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Output feedback control of a skid-steered mobile robot based on the super-twisting algorithm

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ABSTRACT

This paper presents the design and implementation of an output feedback controller based on the super twisting algorithm (STA) that stabilizes the trajectory tracking error of a skid steered mobile robot (SSMR). The control scheme introduces a diffeomorphism based on the mathematical model of the SSMR to transform the original problem into a third order chain of integrators. In this study, the available measurements are the position and orientation of the SSMR. A modified STA working as a step by step differentiator estimates the velocity and acceleration of the mobile robot. Then, a second STA enforces the tracking of a predefined trajectory. Numerical and experimental results comparing the STA with a state feedback controller (SFC) and a first order sliding mode controller (FOSM) justify the control proposal.

1. Introduction

Skid steering mobile robots are recognized as all-terrain robots, since they can be used in non-controlled environmental conditions (Trojnacki, 2015). This kind of vehicles has a robust mechanism structure but they are not equipped with an explicit steering mechanism. As a consequence, the change of orientation in the SSMR produces a lateral slippage between wheels and ground. This feature makes the control solution of SSMRs quite different from classical wheeled mobile robots (Yi, Wang, Song, Jayasuriya, & Liu). The SSMR steers by creating a differential of the forces generated from the actuators located on both sides of the longitudinal axis of the robot. This differential force generates a non-null lateral velocity causing in turn the effect of side skidding. A controller design for a SSMR should be awarded of the differential force generated from the two sides of the robot and therefore the amount of skidding. When a SSMR follows a curved path, its heading is not parallel to the tangent of the curved path because it laterally skids (Wang et al., 2009). The instantaneous center of rotation (ICR) is not fixed as in the case of active steering mobile robots with ideal rolling, that may change continuously. Moreover, in some cases, the ICR may be located outside the robot dimensions along the longitudinal axis causing some kind of instability (Caracciolo, De Luca, & Iannitti, 1999).

The skidding forces produced by the lateral friction on the wheels motivate the control design to consider the knowledge of the SSMR mathematical description of dynamics. This is a strong difference between SSMR and the robots that use an active steering where a kinematic model is considered to design the controller. The wheel/ground interactions provide traction and braking forces affecting the motion stability and maneuverability. The characteristics of the wheel/ground interaction greatly depend on the wheel slip. Because the effect of lateral skidding, velocity constraints occurring in SSMRs are quite different from the ones met in other mobile platforms where wheels are not supposed to skid (Kozlowski & Pazderski, 2004). This fact implies that controlling this robot at the kinematic level is not sufficient and, in general, demands the use of a properly designed control algorithm at the dynamic level too. In Caracciolo et al. (1999), a dynamic model has been derived including the lateral and friction forces. From the modeling point of view, the equilibrium equation of the forces orthogonal to the wheels should be taken into account and this prescribes the use of a dynamic model for control design purposes, instead of a simpler kinematic form. Several control techniques were developed with this dynamic description (Caracciolo et al., 1999; Yi et al.). In some of these approaches, nonlinear controllers are based on the Lyapunov technique such as the ones presented in Angeles, Boulet, Clark, Kovecses, and Siddiqi (2010, chap. 19), and Arslan and Temeltas (2011). Nonlinear controllers using the backstepping concept have been also applied to solve the problem of path tracking of SSMR. Fuzzy controllers, together with a sliding mode (SM) technique have been also designed. In Nazari and Naraghi (2008), a classical fuzzy controller

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was applied where the inputs of this controller were taken as the classical sliding surface used in the first order sliding modes. Even, based on the selection of the sliding surface, the term sliding mode control is not-well defined because a switching function in the controller structure was not introduced. This approach conserves the main disadvantage of any fuzzy controller, that is, the difficult to tuning it, including the membership functions selection and the rules for the inference process. A consequence of the absence of the switching function is the asymptotic convergence of the signal error instead of a finite time convergence (a common feature in the SM) (Utkin, 1992, 2009). Moreover, no proof of convergence was presented. Another attempt to establish a SM regime to control SSMR is introduced in Angeles et al. (2010, chap. 19), where, taking the wheel torques as inputs, a first order sliding modes (FOSM) action is applied to control the yaw angle and its longitudinal velocity. However, as in the previous work, finite-time convergence of the tracking error is not guaranteed. Even when the first order SMs are robust against parametric uncertainties and coupled bounded perturbations, the second order sliding mode approach has been commonly applied to control and estimate second order nonlinear systems (Levant, 1993; Levant & Fridman, 2002).

Second order sliding modes (SOSM) offer attractive performance when the model presents uncertainties or is affected by some kind of bounded perturbations (Levant, 2007). The SOSM preserve the classical features exhibited by FOSM and reduce the undesirable chattering effect. Recently, some new convergence techniques based on non-smooth Lyapunov functions have been introduced to obtain finite time convergence in the problems of state estimation and control (Moreno & Osorio, 2008). These new Lyapunov functions allow an easily tuning of SOSM algorithms such as the STA and the Twisting Algorithm (TA) (Moreno & Osorio, 2012). The STA can be applied as a robust exact differentiator (Levant, 1998), controller (Davila, Moreno, & Fridman, 2009) or state estimator (Davila & Fridman, 2005).

1.1. Contribution

In this paper, the STA is applied to estimate and control the states of a SSMR. The convergence analysis used the second Lyapunov method and the non-smooth Lyapunov functions introduced in Moreno and Osorio (2012) and Gonzalez, Moreno, and Fridman (2012). By the characteristics of the so-called fixed time convergence (the concept of fixed time convergence is addressed in Polyakov & Fridman, 2014) the separation principle applies and two Lyapunov functions are considered in the stability analysis, one for the STA working as a step by step robust differentiator and a second one for the STA working as a controller (Moreno & Osorio, 2012). The proposed strategy was compared in simulations with a controller based on FOSM and a classical SFC. Once the simulation analysis shows and acceptable behavior for the STA fixed-time differentiator and the STA controller, a real implementation is done in a SSMR where the position is obtained by an Optitrack vision system into a delimited area. Although recently in Utkin (2016) some issues about the advantages presented by High Order Sliding (HOSM) were discussed, in particular that the performance presented by HOSM can be obtained with a FOSM, in this work some simulations compared the performance of the STA as a controller against FOSM. The comparative performance showed relevant advantages in the rate of convergence and in the amplitude of the control signal. In the same reference, the author underline the fact that "*The author of the paper would be disappointed if it would be interpreted as an appeal to deny the potential of the HOSM control. The main message of the paper: HOSM is an interesting phenomenon in the sliding mode control theory*". This idea motivated us to test a novel HOSM that actually worked better than the classical FOSM solution.

1.2. Structure of the paper

The following section introduces some mathematical notations that are used throughout the manuscript. Then, in Section 3 the mathematical description of the SSMR is described following the results presented in Caracciolo et al. (1999). Even, when the complete description can be found in the aforementioned reference, we present the complete methodology to clarify how the SSMR can be transformed into a chain of integrators in order to apply the proposed controller. In Section 4 the control strategy based on SOSM is developed presenting all the transformations applied into the SSMR to make feasible the applications of the controller. Also, in this section a step-by-step differentiator is described to obtain two consecutive derivatives needed by the SOSM controller. The proof of convergence for the tracking error is developed in Section 5 using the Lyapunov approach. Numerical simulations are developed in Section 6 in order to have a comparison between the SOSM controller, a FOSM controller and SFC. Once we can prove that the SOSM methodology has some advantages over FOSM and SFC methodologies, the SOSM controller is implemented in a real situation. The results obtained are presented in Section 7. Finally in Section 8 some conclusions are discussed about the obtained results.

2. Notation

- $\mathbb{R}_+ = \{x \in \mathbb{R}: x > 0\}$, $\mathbb{R}_- = \{x \in \mathbb{R}: x < 0\}$ where $\mathbb{R}$ is the set of real numbers;
- $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$, i.e. $\|x\| = \sqrt{x_1^2+\cdots+x_n^2}$ for $x = (x_1, \ldots, x_n)^\top$, $x \in \mathbb{R}^n$
- for a matrix $P \in \mathbb{R}^{n\times n}$, which has the real spectrum, the minimal and maximal eigenvalues are denoted by $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$, respectively;
- if $P \in \mathbb{R}^{n\times n}$, then, the inequality $P > 0$ ($P \geq 0$, $P < 0$, $P \leq 0$) means that P is symmetric and positive definite (positive, semidefinite, negative definite, negative semidefinite);

3. Dynamical model of a SSMR

The dynamical model considered in this paper was developed in Caracciolo et al. (1999) under the following assumptions:

1. Vehicle speed below 10 km/h.
2. Longitudinal wheel slippage neglected.
3. Tire lateral force function of its vertical load.
4. The suspension and tire deformation is neglected.Fig. 1 shows the free body diagram of the SSMR in the x-y phase plane. The main problem working with SSMR is the lateral skidding that is produced when the SSMR is turning (Trojnacki, 2015; Yi et al.; Wang et al., 2009). The forces interacted in each wheel are depicted in Fig. 2. The SSMR obeys the following matrix differential equation:

$$M(q)\frac{d^2}{dt^2}q + R\left(\frac{d}{dt}q\right) + f(q, t) = B(q)\boldsymbol{\tau}. \tag{1}$$

where $q^\top = [X \;\; Y \;\; \theta]$ is the state vector and X and Y are the positions of the center of mass in the $x - axis$ and $y - axis$ respectively and θ is the angle of orientation as it is shown in Fig. 3:

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix}, \quad B(q) = \frac{1}{r}\begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ -c & c \end{bmatrix}, \quad R\left(\frac{d}{dt}q\right) = \begin{bmatrix} F_{rx}\left(\frac{d}{dt}q\right) \\ F_{ry}\left(\frac{d}{dt}q\right) \\ M_r\left(\frac{d}{dt}q\right) \end{bmatrix}$$