Electric Power Systems Research 154 (2018) 311–318

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

New analytical method for estimating mean life of electric power equipment based on complete and right-censored failure data

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ARTICLE INFO

Article history:
Received 18 May 2017
Received in revised form 3 August 2017
Accepted 31 August 2017

Keywords:
Power equipment
Weibull distribution
Parameters estimation
Lifetime
Maximum likelihood
Censored data

ABSTRACT

Analysis based on application of two-parametric Weibull distribution is commonly used to characterize power equipment life distributions in the presence of censored data. Numerous studies have focused on enhancing the accuracy of statistical parameter estimation. However, there is still no straightforward and rigorous analytical method of estimation of the Weibull parameters and the mean life. In this work, a new method for mean life evaluation of power system equipment, based on the two-parameter Weibull distribution, is presented for complete and right-censored failure data. Classical maximum likelihood estimation (MLE) is employed for determination of distribution parameters. However, an estimator based on asymptotic expansions is proposed, overpassing the disadvantages of MLE-based methods that employ numerical or graphical techniques. High accuracy of the proposed method with respect to other estimators is also shown by analyzing two right-censored lifetime data sets with different sample sizes for three types of power equipment. The efficiency and accuracy of the proposed analytical method find their strengths in the analytically obtained closed-form expressions for the distribution parameters.

1. Introduction

Ageing of power system equipment is one of the major issues that has emerged in recent years [1]. According to the survey [2] conducted by Cigre Working Group, since 2008 much of the installed equipment has exceeded its design lifetime and are considered as old equipment. This results in increasing risks of interruptions for customers and it can lead to million dollar losses for utilities and individual users. In addition, replacement times of equipment can jeopardize the reliable operation of the power network, if outages suddenly and unexpectedly appear. Therefore, it is important to properly predict the lifetime of power equipment, so that the number of untimely failures can be reduced such that reliability and reinvestment costs prevail.

The failures of power system components can be classified into two types: repairable and end-of-life failures [3]. The data of the second type of failure is required for the estimation of lifetime of these devices. However, the availability of end-of-life failure data is scarce due to the fact that most of the power equipment have long average lifetimes and low failure rates. In fact, what is available is an excessive amount of "censored" data. These data contain incomplete information about the lifetime of equipment, since they are actually survival times of components, which have not yet experienced end-of-life failures. The presence of censored data may complicate the accuracy of statistical procedures used in lifetime data analysis.

A lot of research has been already performed to estimate the mean life of power equipment in the presence of censored data. Some studies have particularly focused on enhancing the accurate estimation of statistical parameters. One can notice in these research efforts that several distributions have been used to characterize end-of-life failures, including two-parameter Weibull distribution [3–10], normal distribution [5], log-normal distribution [11], three-parameter Weibull distribution [12], and generalized exponential distribution [13,14]. However, the two-parameter Weibull distribution, which is defined by the shape and scale parameters, is a commonly used model in reliability and lifetime data analysis [15–18].

Mean life estimation for power equipment is mainly based on the two-parameter Weibull distribution, applying traditional and numerical methods of parameter estimation. Normal and Weibull distributions have been applied to estimate the mean life and standard deviation of a power equipment group in [5]. The author applied the well-known least squares method to estimate the Weibull parameters from a survival probability. A gradient descent technique has been used for numerical estimation, with initial data

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http://dx.doi.org/10.1016/j.epsr.2017.08.042
0378-7796/

Abbreviations

Abbreviations

AM	analytical method
GED	generalized exponential distribution
GEM	generalized exponential distribution method
HM	hybrid method
LSR	least squares regression
MLE	maximum likelihood estimation
NR	Newton–Raphson Method
BCTC	British Columbia Transmission Corporation

Variables

$f(t\|\alpha, \beta)$	conditional probability density
$R(t\|\alpha, \beta)$	conditional survival function
$L(\alpha, \beta)$	log-likelihood function

Symbols

α	Weibull scale parameter
β	Weibull shape parameter
α^*	calculated scale parameter
β^*	calculated shape parameter
β_0	initial value for the shape parameter in the NR method
δ_k	failure-censoring label
μ	mean life
n	number of failed components
N	sample size
σ	standard deviation
t_k	kth failure-censoring time

obtained from a normal distribution model. The author states: "It is difficult to select an appropriate resolution algorithm for such a set of nonlinear equations. If a numerical algorithm is used, there is no easy criterion to select initial values to arrive at the rational one in the multiple solutions". A modified version of the least squares method for parameter estimation is presented in [8]. However, the shape parameter is numerically estimated using a specialized software in this work. The proposed method in [5] is called "hybrid method" in [6]. The so-called data-analytic approach is proposed to estimate the Weibull parameters and mean life in case of right-censored data. This method is "relatively simpler and faster than the hybrid method", mainly because within this approach the parameters are fitted by using linear regression, based on empirical survival functions and the least squares technique. The method developed in [7] is more robust than the previous ones, since Weibull parameter estimation is based on the maximum likelihood approach. However, the parameters are fitted numerically by a direct maximization of the likelihood function via the Newton-Raphson algorithm. A comparison of the maximum likelihood and the median rank regression methods for estimating transformer lifetime using the Weibull distribution is presented in [9]. In the maximum likelihood estimation method, the shape parameter is fitted numerically by applying the Newton–Raphson algorithm. The impact of survival data on the accuracy of lifetime models is analyzed in [10]. The maximum likelihood method is applied for the parameter estimation. The default function "wblfit", from the statistical toolbox of Matlab, has been chosen to estimate the corresponding Weibull parameters.

Although the above references have made important contributions to the research field of power equipment condition evaluation, the traditional parameters estimation techniques have some disadvantages which have been discussed in the specialized literature. For example, probability plot methods are straightforward, but precise estimations are not attained [5,19]. Methods based on regression analysis are linear estimators that put a large weight on the extreme observations having large variance [20]. MLE is considered to be one of the most robust parameter estimation techniques. It can handle survival and interval data better than the least squares method particularly when dealing with heavily censored data sets that contain few exact observed data. However, MLE has the inconvenience that the corresponding likelihood equations need to be solved numerically. Hence, issues of low convergence rate and efficiency in the iterative steps need to be addressed, which can be particularly difficult in cases of censored data [21].

Recent research [22–26] have been focused on obtaining new efficient numerical and statistical inference methods in order to eliminate the disadvantages of traditional parameter estimation methods in the field of lifetime analysis. Nevertheless, the Weibull parameter estimation problem continues to be important since there is still interest in developing general methods of parameter estimation for a wider range of applications.

An asymptotic method to estimate the Weibull distribution parameters for complete and right-censored data is developed in this work. The solution to the likelihood equation is obtained as a power series in a small parameter. It is shown through several examples that this method provides excellent precision. Noteworthy, our proposed solution includes new closed-form analytical formulae for the estimation of the shape and scale parameters of the Weibull distribution.

The paper is organized as follows. The problem is stated in Section 2: (1) The maximum likelihood equations are reduced to a single dimensionless nonlinear likelihood equation for the shape parameter. (2) The existence and uniqueness of solution to this equation is discussed. The asymptotic method of solution to the likelihood equation is developed in Section 3. The solution is obtained as an asymptotic expansion with an appropriately estimated residual term. Three different study cases are considered in Section 4 to assess the performance of the developed method. Weibull distribution parameters and mean life of power equipment are calculated for three extreme scenarios. The numerical and analytical results are analyzed in Section 5.

2. Model

The development of the model considers that the lifetime set (end-of-life or censored data) t_k, where $k = \overline{1, N}$, is numbered in an ascendant order, i.e., $0 < t_1 \leq t_2 \leq \ldots \leq t_N$, where N is the total number of components. Let δ_k denote a variable indicating t_k as the end-of-life or censored time such that:

$$\delta_k = \begin{cases} 1 & \text{if } t_k \text{ is the age of a failed component,} \\ 0 & \text{if } t_k \text{ is the age of a survived component.} \end{cases}$$

So, the total numbers of failed components are:

$$n = \sum_{k=1}^{N} \delta_k \tag{1}$$

and the numbers of surviving components are $N - n$, respectively. It is assumed that the lifetime data set obeys the Weibull distribution with a probability density function:

$$f(t|\alpha, \beta) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-(t/\alpha)^{\beta}} \tag{2}$$

and the corresponding survival function is given by:

$$R(t|\alpha, \beta) = \int_t^{\infty} f(\tau|\alpha, \beta) d\tau = e^{-(t/\alpha)^{\beta}}. \tag{3}$$