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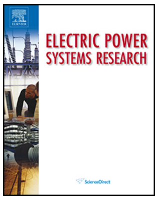

A full frequency dependent line model based on folded line equivalencing and latency exploitation

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ABSTRACT

This work proposes to improve the numerical performance in the rational modeling of full frequency dependent transmission lines. In this paper, latency is used to improve the numerical efficiency of time-domain implementation of the pole-residue representation of the nodal admittance matrix in the case of frequency dependent network equivalents. The idea of latency is to use different time-steps for distinct poles, i.e., fast poles are solved using a small time-step while slower poles may be solved using larger time-steps. For the separation of the slow and fast dynamics, a technique named Multiple Companion Networks (MCN) has been developed. The technique has been applied to transient tests of a transmission line modeled with a rational model. The results obtained indicated that a gain in numerical efficiency is possible without compromising accuracy.

1. Introduction

Electromagnetic transient (EMT)-programs allowed for a rapid and wide development of time-domain models for linear, nonlinear and frequency dependent devices. One main characteristic of such approach is to rely on the use of a single time-step for the solution of the whole network. This is often a waste of computational resources since it may be the case where only a small part of the network has to be solved using a small time step, whereas a large part of the network could be solved with a larger time step. Thus a suboptimal numerical performance is found leading to an unnecessary large simulation time.

One possibility to overcome the unnecessary simulation time and to improve the numerical performance of the models involved is to rely on the concept of latency exploitation. In latency [1], the network is divided into at least two sub-networks. The first one, associated with the fastest time-constants, is solved using a small time-step, while a larger time-step is used to solve the other part of the network. Naturally, the number of network subdivisions can increase leading to several sub-networks each being solved with a given time-step. For this reason, latency exploitation may also be called multirate simulation.

The development of numerically accurate and efficient methods for the rational approximation of frequency responses such as the so-called vector fitting routine [2–4], the frequency-partitioning fitting [5] or the matrix pencil [6,7] allowed the realization of full frequency dependent overhead lines and underground cable models in phase coordinates in EMT-simulations [8,9]. One drawback of such models which are based on the method of characteristics (MoC) is that the time-step must be smaller than the fastest modal time-delay which requires a rather accurate procedure for this identification [10]. Furthermore, inaccurate interpolation schemes between the simulation time-step and the modal traveling time might lead to numerical instabilities [11]. This scenario is more important for shorter lines where a very small time-step might be used and the interpolation of the modal time-delay might reduce the numerical efficiency of a line model based on MoC. An alternative to overcome this limitation is to use a rational approximation of the nodal admittance matrix associated with the transmission line. However, as pointed out in [12], the direct fitting of the nodal admittance matrix is not suitable for a general transmission line model as the smallest eigenvalues at the low-frequency range may not be properly fitted leading to inaccurate responses. The limitation associated with the direct fitting can be overcome with the use of the folded line equivalent (FLE) model as also shown in [12].

In [13], the authors proposed to exploit latency in a frequency dependent network using a rational approximation of the nodal admittance matrix developing the Multiple Companion Network (MCN) approach. In this work the formulation of the MCN based

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on a model decomposition using the FLE has been extended. It was found that the earlier approach led to rather inaccurate results for the time responses and it was needed to rearrange how latency is exploited in the MCN.

The paper is organized as follows: Section 2 summarizes the application of FLE to the modeling of an overhead line. Section 3 explains the idea behind latency exploitation and the use of MCN and how some improvements can be achieved using this formulation. Section 4 describes how a modified MCN can be integrated in the time-step loop to allow an efficient implementation of the proposed model in an EMT-type of program. Section 5 illustrates the main idea of the paper, i.e., to exploit latency considering a FLE representation of an overhead line. The main conclusions of this work are presented in Section 6.

2. Folded line equivalent

Consider a transmission line with n phases, the nodal admittance matrix $\mathbf{Y}_n$ relating the injected currents and the node voltages is given by (1) in the frequency domain,

$$\mathbf{Y}_n = \begin{bmatrix} \mathbf{Y}_s & \mathbf{Y}_m \\ \mathbf{Y}_m & \mathbf{Y}_s \end{bmatrix} \tag{1}$$

where $\mathbf{Y}_s$ and $\mathbf{Y}_m$ are $n \times n$ block matrices defined by

$$\begin{aligned} \mathbf{Y}_s &= \mathbf{Y}_c \cdot \left(\mathbf{I} + \mathbf{H}^2\right) \cdot \left(\mathbf{I} - \mathbf{H}^2\right)^{-1} \quad \textit{and} \\ \mathbf{Y}_m &= -2\mathbf{Y}_c \cdot \mathbf{H} \cdot \left(\mathbf{I} - \mathbf{H}^2\right)^{-1} \end{aligned} \tag{2}$$

where $\mathbf{I}$ is an $n \times n$ identity matrix, $\mathbf{Y}_c = \mathbf{Z}^{-1}\sqrt{\mathbf{Z} \cdot \mathbf{Y}}$ is the characteristic admittance matrix, $\mathbf{H} = \exp(-\ell\sqrt{\mathbf{Z} \cdot \mathbf{Y}})$ is the propagation matrix also known as voltage deformation matrix and ℓ is the line length.

In EMT-programs, the line is not directly represented using a rational approximation of $\mathbf{Y}_n$ as there is a large spread of the eigenvalues throughout the frequency range of interest. The method of characteristics (MoC) is used instead which implies in a rational approximation of $\mathbf{Y}_c$ and $\mathbf{H}$. While the former is basically a smooth function in the frequency domain, the latter may present some oscillations that demand the use of several travelling time-delays to achieve an accurate fitting, for instance see [14,8]. This procedure also demands the use of a more elaborate interpolation scheme to avoid numerical instability [11]. A possibility to overcome this limitation is to rely on the so-called idempotent decomposition of $\mathbf{H}$ [15]. However, as pointed out in [16], there might be some configurations where the overall accuracy of the rational fitting of the idempotent is low. Alternatively, one may apply a rational approximation not to $\mathbf{Y}_n$ but to a "folded" version of it. In this case, the idea is to use a linear transformation to allow a distinct grouping in $\mathbf{Y}_n$ in such way that the rational approximation has a lower order and a higher accuracy. This procedure shares some similarities with the usage of a mode-revealing transformation matrix to improve the observability of small eigenvalues at the lower frequency range [17]. Thus, the nodal admittance matrix is written as

$$\mathbf{Y}_n = \mathbf{K} \cdot \begin{bmatrix} \mathbf{Y}_{oc} & \mathbf{0} \\ \mathbf{0} & \mathbf{Y}_{sc} \end{bmatrix} \cdot \mathbf{K}^{-1} \tag{3}$$

where $\mathbf{Y}_{oc} = \mathbf{Y}_s + \mathbf{Y}_m$ is the admittance associated with the open circuit current response and $\mathbf{Y}_{sc} = \mathbf{Y}_s - \mathbf{Y}_m$ stands for the admittance related to the short circuit current response, and

$$\mathbf{K} = \begin{bmatrix} \mathbf{I} & \mathbf{I} \\ \mathbf{I} & -\mathbf{I} \end{bmatrix} \tag{4}$$

where $\mathbf{I}$ is the same as before. This formulation was proposed in [12] and received the name folded line equivalent (FLE). It has some interesting characteristics, the matrices to be fitted have half the dimension of the original admittance matrix, i.e., $\mathbf{Y}_n$ has a dimension $2n \times 2n$ while both $\mathbf{Y}_{oc}$ and $\mathbf{Y}_{sc}$ are $n \times n$ matrices. Furthermore, the ratio between the largest and smallest eigenvalues in both $\mathbf{Y}_{oc}$ and $\mathbf{Y}_{sc}$ reduces considerably when compared with the ones found in $\mathbf{Y}_n$.

The rational approximation of $\mathbf{Y}_{oc}$ and $\mathbf{Y}_{sc}$ is carried out independently. In this work, the use of the so-called vector fitting routine [2–4] was chosen although other identification methods such as the frequency-partitioning fitting [5] or the matrix pencil [6,7] could be used instead. Typically, a rational approximation of a nodal admittance matrix $\mathbf{Y}_n$ leads to the following

$$\mathbf{Y}_n \approx \sum_{m=1}^{N} \frac{\mathbf{R}_m}{s - p_m} + \mathbf{R}_0 \tag{5}$$

where N is the number of poles (pre-defined) to be used, p_m is a set of common poles, either real or in complex conjugates, $\mathbf{R}_m$ is the residue matrix, and $\mathbf{R}_0$ is a real-valued matrix associated with the behavior of $\mathbf{Y}_n$ at an infinite frequency. In this case, $\mathbf{R}_0 = \Re\left(\mathbf{Y}_c(\infty)\right) = \mathbf{G}_c(\infty)$. For the matrices $\mathbf{Y}_{oc}$ and $\mathbf{Y}_{sc}$, it is required that both tend to the characteristic admittance in the high frequency range, i.e., both models must be asymptotically correct. Therefore, the fitting is carried out using (6),

$$\begin{aligned} \bar{\mathbf{Y}}_{oc} &= \mathbf{Y}_{oc} - \mathbf{G}_c(\infty) \approx \sum_{m=1}^{N} \frac{\mathbf{R}_{oc_m}}{s - p_{oc_m}} \\ \bar{\mathbf{Y}}_{sc} &= \mathbf{Y}_{sc} - \mathbf{G}_c(\infty) \approx \sum_{m=1}^{M} \frac{\mathbf{R}_{sc_m}}{s - p_{sc_m}} \end{aligned} \tag{6}$$

It is assumed that both functions to be fitted, i.e., $\bar{\mathbf{Y}}_{oc}$ and $\bar{\mathbf{Y}}_{sc}$, are strictly proper and there is no a priori relationship between the number of poles used in the fitting of either function. Even though only stable poles are used in the fitting of $\mathbf{Y}_{oc}$ and $\mathbf{Y}_{sc}$, the passivity of both approximations must be enforced, as some of the eigenvalues might present a negative real part in the frequency band. Thus, a post-processing scheme based on residue perturbation is applied to $\mathbf{Y}_{oc}$ and $\mathbf{Y}_{sc}$ to ensure passivity [18].

It is noteworthy a great feature of this approach that the accurate eigenvalues obtained in the fitting process allows to convert the FLE realization back to phase coordinates via (3) with small error-magnifications to be applied in simulations with arbitrary terminal conditions [12].

3. Latency exploitation & Multiple Companion Networks

Latency was first applied by researchers in the area of VLSI (Very Large System Integration) simulation [19]. The attempts were based on the waveform relaxation method which is an iterative method capable of exploiting time-domain latency to reach significant speed gains without sacrificing accuracy. It is based on the Gauss-Seidel and Gauss-Jacobi relaxation methods applied to the numerical integration of DAEs (differential algebraic equations).

Latency was first explored in power system problems, also within the context of the waveform relaxation method, for transient stability simulation [20] and later for the solution of electromagnetic transient simulations using multiple time-steps [21,22]. Also, latency has been exploited in an HVDC converter network where the converter bridges have been simulated with a small time step while the ac sources, transformers and dc line have been simulated with a larger time step. Results have shown that accuracy has been maintained while simulation time can be significantly reduced [23].