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Electrical network dynamic models with application to modal analysis of harmonics

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ABSTRACT

This paper describes two network models which allow several harmonic and electromagnetic transient analysis, being, therefore, more flexible than conventional ones. In general, these models can be used to perform time simulation, frequency scan and modal analyses. The system matrices are assembled for an industrial system example. Harmonic problems are proposed and solved using a non-conventional analysis with the described models. Results on a large-scale power system regarding computational performance are also included.

1. Introduction

The proposed methodology utilizes two electrical network-modeling techniques, named descriptor systems [1] and **Y**(s) matrix [2–4], that allow electrical network analyses over the entire complex plane s instead of just over the imaginary "$j\omega$" axis [5]. In this expanded domain, modal analysis can be performed, providing an important set of structural system information that is hard to obtain using time simulation or frequency response methods [5]. The information provided by modal analysis includes the natural oscillation modes (system poles), identification of equipment that more heavily participate in these modes, modal resonance sensitivities, etc. This structural information set has been used to solve harmonic problems [1], electromagnetic transient analysis [4] and build network dynamic equivalents [6]. It must be pointed out that with the increase of the renewable energy sources, such as wind farms, in the power systems, resonance sensitivity analysis has become an important tool for identifying and solve harmonic resonance problems [7–11]. This timely subject is exploited in the application example. The main contributions of this paper are:

- Presentation of the whole set of component models, including the descriptor system modeling of three-winding transformers, transmission lines using cascaded RLC-π circuits and voltage sources, not previously presented in the literature. Some examples using the two proposed modeling techniques, including symbolic matrices of a test system, give an important tutorial aspect to this paper.
- A comparison between the descriptor system and **Y**(s) network modeling techniques.
- An application in harmonics consisting in a methodology for shifting a set of system poles to more suitable locations in the complex plane to reduce harmonic voltage distortions more straightforward than previous techniques [1,2,10,11].
- Considerations on the computational performance of the two modeling techniques applied to a large-scale power system.

2. Network modeling techniques for modal analysis

Despite of the advantages of using modal analysis, it has been only moderately used in power system studies considering the electrical network dynamics. This fact may be associated with the difficulties faced when using conventional state space techniques for modeling the dynamics of large RLC networks of generic topology [1]. These difficulties are eliminated when using the proposed techniques explained in the following sub-sections. The modeling

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of the basic components of electrical networks is presented in the sequence.

2.1. Descriptor system

The electrical network modeling by descriptor system is accomplished by an augmented differential-algebraic equation (DAE) system, as shown in (1) and (2).

$$\mathbf{T}\,\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t) \tag{1}$$

$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t) \tag{2}$$

where $\mathbf{x}$ is the vector of system variables, including state and algebraic ones, $\mathbf{u}$ and $\mathbf{y}$ are the input and output vectors. In this proposed modeling, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{T}$ are very sparse matrices that include the constant coefficients of the linear system equations.

In the particular case of the proposed modeling, the augmented state vector is composed by all inductive currents, all capacitors voltages and some additional variables, as will be seen in the sequence. State variable redundancies caused, for instance, by nodes having only inductors or meshes having only capacitors do not need to be eliminated [1]. Matrix $\mathbf{T}$ is diagonal, having zeros for the lines corresponding to algebraic equations and non-zero elements for the lines corresponding to differential equations. The dynamic behavior of the electrical network is described by the equation system produced by the differential and algebraic equations of the electrical components together with those proceeded from the Kirchhoff's current law (KCL) applied to each circuit node. In the following subsections, the electrical component equations will be developed while the matrices for a simple test system including the KCL equations are presented in Section 3.

2.1.1. RLC series branch

A RLC series branch connected between the nodes (buses) k and j is described by:

$$v_k - v_j = R\,i_{kj} + L\,\frac{di_{kj}}{dt} + v_C \tag{3}$$

$$C\,\frac{dv_C}{dt} = i_{kj} \tag{4}$$

where v_k and v_j are the voltages of nodes k and j, respectively, i_{kj} is the branch current and v_C is the capacitor voltage. These equations hold when there is no inductor ($L=0$) in the branch. When there is no capacitor ($C \to \infty$), one may use only (3) with $v_C = 0$, excluding (4).

2.1.2. RLC parallel branch

In this case the equations are:

$$\frac{v_C}{R} + i_L + C\,\frac{dv_C}{dt} = i_{kj} \tag{5}$$

$$L\,\frac{di_L}{dt} = v_C \tag{6}$$

$$v_C = v_k - v_j \tag{7}$$

The symbols v_k, v_j, i_{kj} and v_C have the same meaning of the RLC series branch. These equations hold when there is no capacitor ($C=0$) in the branch. For the case of $L \to \infty$, one may use only (5) and (7) with $i_L = 0$, excluding (6).

2.1.3. Voltage source

A voltage source v_f with internal resistance R_f and inductance L_f connected between the nodes k and j is described by:

$$v_k - v_j = v_f - L_f\,\frac{di_f}{dt} - R_f\,i_f \tag{8}$$

where v_k and v_j have already been defined and i_f is the current provided by the voltage source.

2.1.4. Transmission line

In the descriptor system formulation, the transmission lines can be modeled by cascaded RLC-π circuits as shown in Fig. 1, connecting the physical nodes k and j. Between these nodes, there are internal fictitious nodes used to assemble the model equations. The internal nodes were numerated from 1 to n, being a generic node denoted by m.

Using the node equation $\sum i = 0$ for the node 1, one obtains:

$$\frac{C}{2}\,\frac{dv_1}{dt} = -i_{1,2} + i_{k,1} \tag{9}$$

Using the mesh equation $\sum v = 0$ for the generic loop of nodes m and $m+1$, one obtains:

$$L\,\frac{di_{m,\,m+1}}{dt} = -R\,i_{m,\,m+1} + v_m - v_{m+1}\,, \quad m = 1,\ n-1 \tag{10}$$

The node equation $\sum i = 0$ applied to the generic node m and the last node n yields, respectively:

$$C\,\frac{dv_m}{dt} = i_{m-1,\,m} - i_{m,\,m+1}\,, \quad m = 2,\ n-1 \tag{11}$$

$$\frac{C}{2}\,\frac{dv_n}{dt} = i_{n-1\,,\,n} - i_{\,n,\,j} \tag{12}$$

Finally, the interface equations, relating the internal node voltages of the model with the network voltages at nodes k and j are given by:

$$v_k - v_1 = 0 \tag{13}$$

$$v_j - v_n = 0 \tag{14}$$

2.1.5. Three-winding transformer

Fig. 2 presents the positive sequence diagram of a three-winding transformer, with its variables in pu, connecting the physical nodes k, j and l of an electrical network. The symbols m_1, m_2 and m_3 correspond to the values of the winding taps. Depending on the three phase connections of the windings, there may be phase-shifts among windings taken into account by complex taps [12]. Parameters $R_{T_1}, L_{T_1}, R_{T_2}, L_{T_2}, R_{T_3}$ and L_{T_3} are winding resistances and leakage inductances of the windings connected to buses k, j and l, respectively. The considered voltages and currents are also indicated in the circuit.

The voltage drop $v_{1,4}$ between nodes 1 and 4 is given by:

$$L_{T_1}\,\frac{di_{1,4}}{dt} = -R_{T_1}\,i_{1,4} + v_{1,4} \tag{15}$$

where:

$$v_{1,4} = v_1 - v_4 \tag{16}$$

In addition:

$$v_1 = \left(1/m_1\right)\,v_k \tag{17}$$

Substituting (17) in (16), one obtains:

$$v_{1,4} = \frac{v_k}{m_1} - v_4 \tag{18}$$

Substituting (18) in (15), one obtains:

$$L_{T_1}\,\frac{di_{1,4}}{dt} = -R_{T_1}\,i_{1,4} + \frac{v_k}{m_1} - v_4 \tag{19}$$

Analogously, one has:

$$L_{T_2}\,\frac{di_{2,4}}{dt} = -R_{T_2}\,i_{2,4} + \frac{v_j}{m_2} - v_4 \tag{20}$$