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Uniqueness of the power flow solutions in low voltage direct current grids

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ABSTRACT

Low voltage direct current (LVDC) is a promising technology for future power distribution grids and smart grids applications. Power flow in these grids is a non-linear problem just as its counterpart AC. This paper demonstrates that, unlike in AC grids, convergence and uniqueness of the solution can be guaranteed in this type of grid under well defined practical considerations. The result is neither a linearization nor an approximation, but an analysis of the set of non-linear algebraic equations, which is valid for any LVDC grid regardless its size, topology or load condition. Computer simulation corroborates the theoretical analysis.

1. Introduction

1.1. Motivation

Low voltage direct current (LVDC) is a promising technology for urban distribution systems, micro-grids, data centers, traction power systems and shipboard power systems [1]. It presents advantages in terms of reliability, efficiency, controllabiliy, power density and loadability [2,3].

An LVDC grid consists of a bidirectional AC/DC converter placed in the main substation to which it is connected different loads and generators as depicted in Fig 1 . Different elements can be connected to an LVDC grid such as renewable energy resources, energy storage, electric vehicles and controlled loads. These elements are integrated to the grid through a power electronic converter (i.e. a constant power terminal). Consequently, the model of the LVDC grid is non-linear and requires a power flow study.

The existence and uniqueness of the solution are, obviously, *sine qua non* conditions for rigorous analysis of the stationary state of a grid and for determining an equilibrium point in small signal stability studies [4–6]. These are characteristics of the set of algebraic equations and not of the method used to find a solution. However, it is often difficult to determine if a solution of a set of non-linear equations, such as those of the power flow, is unique. A non-linear problem could give several solutions; in some cases, the solution may not even exist and hence uniqueness must not be taken for granted.

1.2. DC power flow vs power flow in LVDC grids

It is important to emphasize that power flow in LVDC grids is different from the well known DC power flow in conventional power systems [7]. The former is a power flow in a grid which is actually DC and incorporates constant power terminals; while the latter is a linearization of the power flow equations in AC grids which, due to a pedagogic analogy, is named in this way. The most important difference is that LVDC power flow equations define in a non-linear, non-affine and not convex space due to the presence of power electronic converters with constant power controls. In addition, the state variables are voltages and not angles which actually does not exist in LVDC grids.

1.3. Brief state of the art

There is an increasing interest in LVDC grids and related subjects such as DC microgrids and DC distribution. Several studies have been done about the feasibility of these technologies. For instance, [1] presented a complete description of the potentialities of LVDC grids as well as their challenges. Potential pathways for increased use of DC technology in buildings was considered in [3]. A more practical approach was presented in [2] where a case study for a large distribution network was considered. These studies show that LVDC grids are interesting not only from the research/theoretical point of view but also from the practical point of view.

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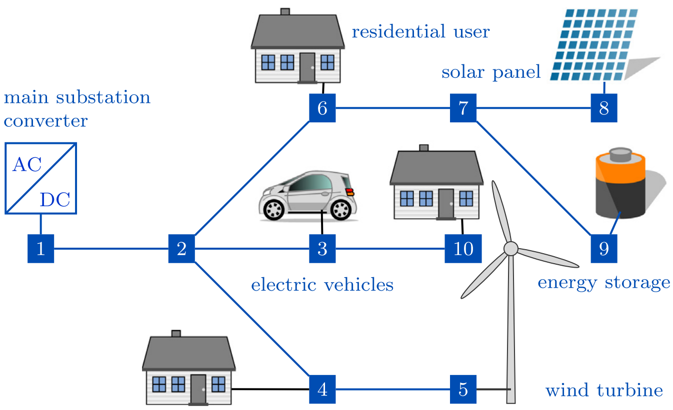

Fig. 1. Example of an LVDC system for urban area applications.

Power flow analysis in LVDC grids has been presented as an extension to well known methodologies for AC grids such as Newton–Raphson or Gauss–Seidel [8]. Power flow sensitivities have been also studied in [9]. However, available studies in the literature are based on numerical performance but there are no theoretical studies about uniqueness of the solution. In these studies, uniqueness is taken for granted without mathematical demonstration in spite of the fact that a non-linear problem could give several solutions. To the best of the author's knowledge, this problem has not been addressed in LVDC grids. The problem has been recently studied in AC power distribution [10]. However, more research is required in this direction.

1.4. Contribution and scope

This paper demonstrates the existence and uniqueness of the solution of the power flow in LVDC grids. This result is general since: (1) it is independent of the numerical method, (2) it is independent of size and load condition of the LVDC grid and (3) it is valid for any topology of the LVDC grid. A computational simulation demonstrates the theoretical analysis using a successive approximation method.

Comparisons of the computational performance of different algorithms are beyond of the scope of this paper in order to maintain the generality of the main result. Computational performance depends on many factors such as the implementation of the algorithm, programming language and size of the grid.

1.5. Organization of the paper

The paper is organized as follows: Section 2 presents the basic formulation of the power flow in LVDC grids from a practical context. Next, Section 3 demonstrates the main theoretical result followed by numerical simulations in Section 4. Finally conclusions and references are presented.

2. Power flow in LVDC grids

The lack of reactive power and angles in LVDC grids allows some simplifications of the mathematical formulation. Nodes are classified according to the type of control, namely: constant voltage, constant power and constant resistance. Constant voltage terminals include the main substation converter and any converter along the grid which can maintain the voltage. Other converters in the grid must be represented as constant power terminals. These include renewable energy resources, energy storage devices and controlled loads, among others. Constant resistance terminals are linear loads as well as step nodes (i.e. nodes without generation or load). Droop controls can be considered as a linear combination of a constant power and a constant resistance terminal [11,12].

2.1. Mathematical formulation

Let us consider an LVDC grid as a set of nodes represented by $\mathcal{N} = \{1, 2, \ldots, N\}$, which in turns is subdivided into three nonempty and disjoint subsets $\mathcal{N} = \{\mathcal{V}, \mathcal{R}, \mathcal{P}\}$ according to the type of terminal, namely: constant voltage ($\mathcal{V}$), constant resistance ($\mathcal{R}$) and constant power $\mathcal{P}$. There is usually only one constant voltage terminal but the methodology can be applied to a more general case with multiple voltage-controlled terminals. Branches are represented as a set $\mathcal{E} = \mathcal{N} \times \mathcal{N}$ with an associated constant resistance.

Nodal voltages and currents are related by the admittance matrix $G \in \mathbb{R}^{\mathcal{N}\times\mathcal{N}}$ as follows:

$$\begin{pmatrix} I_\mathcal{V} \\ I_\mathcal{R} \\ I_\mathcal{P} \end{pmatrix} = \begin{pmatrix} G_{\mathcal{V}\mathcal{V}} & G_{\mathcal{V}\mathcal{R}} & G_{\mathcal{V}\mathcal{P}} \\ G_{\mathcal{R}\mathcal{V}} & G_{\mathcal{R}\mathcal{R}} & G_{\mathcal{R}\mathcal{P}} \\ G_{\mathcal{P}\mathcal{V}} & G_{\mathcal{P}\mathcal{R}} & G_{\mathcal{P}\mathcal{P}} \end{pmatrix} \cdot \begin{pmatrix} V_\mathcal{V} \\ V_\mathcal{R} \\ V_\mathcal{P} \end{pmatrix} \quad (1)$$

In this case, $V_\mathcal{V}$ is known and $I_\mathcal{R}$ is given by (2)

$$I_\mathcal{R} = -D_{\mathcal{R}\mathcal{R}} \cdot V_\mathcal{R} \quad (2)$$

where $D_{\mathcal{R}\mathcal{R}}$ a diagonal matrix that includes admittances of constant resistance terminals. That is, for each constant resistance terminal $d_{kk} = 1/r_{load}$. Notice this matrix can be singular (e.g. in the case of step nodes). Eq. (2) is used to reduce the size of the set of algebraic equations:

$$V_\mathcal{R} = -(D_{\mathcal{R}\mathcal{R}} + G_{\mathcal{R}\mathcal{R}})^{-1} \cdot (G_{\mathcal{R}\mathcal{V}} \cdot V_\mathcal{V} + G_{\mathcal{R}\mathcal{P}} \cdot V_\mathcal{P}) \quad (3)$$

Power-controlled terminals are associated with the following non-linear equation

$$P_\mathcal{P} = diag(V_\mathcal{P}) \cdot I_\mathcal{P} \quad (4)$$

Which in turn can be written as follows

$$P_\mathcal{P} = diag(V_\mathcal{P}) \cdot (J_\mathcal{P} + B_{\mathcal{P}\mathcal{P}} \cdot V_\mathcal{P}) \quad (5)$$

with

$$J_\mathcal{P} = (G_{\mathcal{P}\mathcal{V}} - G_{\mathcal{P}\mathcal{R}} \cdot (D_{\mathcal{R}\mathcal{R}} + G_{\mathcal{R}\mathcal{R}})^{-1} \cdot G_{\mathcal{R}\mathcal{V}}) \cdot V_\mathcal{V}$$

$$B_{\mathcal{P}\mathcal{P}} = G_{\mathcal{P}\mathcal{P}} - G_{\mathcal{P}\mathcal{R}} \cdot (D_{\mathcal{R}\mathcal{R}} + G_{\mathcal{R}\mathcal{R}})^{-1} \cdot G_{\mathcal{R}\mathcal{P}}$$

Therefore, the state of the LVDC grid can be completely established by solving (6).

$$V_\mathcal{P} = B_{\mathcal{P}\mathcal{P}}^{-1} \cdot (diag(V_\mathcal{P})^{-1} \cdot P_\mathcal{P} - J_\mathcal{P}) \quad (6)$$

In order to analyze (6), let us define a map $T : \mathbb{R}^\mathcal{P} \to \mathbb{R}^\mathcal{P}$ as given in (7):

$$T(V_\mathcal{P}) = B_{\mathcal{P}\mathcal{P}}^{-1} \cdot (diag(V_\mathcal{P})^{-1} \cdot P_\mathcal{P} - J_\mathcal{P}) \quad (7)$$

Notice that T is a non-linear map and as aforementioned, uniqueness of the solution must not be taken for granted.

2.2. Practical considerations

Let us consider the following few practical assumptions:

1 the graph is connected (i.e. there are no islands in the feeder)
2 there is at least one constant power terminal and one constant voltage terminal
3 feasible voltages remain in a given interval

$$0 < v_{\min} \le v \le v_{\max} \quad (8)$$

4 short circuit currents are higher than normal operation currents for all constant power terminals.