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Midpoint-radius interval-based method to deal with uncertainty in power flow analysis

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ABSTRACT

This paper presents a novel method based on midpoint-radius interval arithmetic to deal with uncertainties in the power flow problem. The proposed technique aims at finding a balance between accuracy and computational efficiency. It relies on an original decoupling of the interval power flow equations into midpoint and radius parts. This representation allows avoiding the factorisation of an interval Jacobian matrix. Moreover, the proposed formulation is combined with an optimisation problem in order to prevent overestimation of the interval solution while preserving uncertainty. The proposed technique proves to be more efficient than existing approaches based on interval and affine arithmetic and as accurate as the conventional Monte Carlo method.

1. Introduction

1.1. Motivations

Uncertainty is inherent to any physical systems. This is particularly true for power systems, where uncertainty can have several causes, e.g., imprecise demand forecast, price variability, renewable energy generation, economic growth, industry placement, and line aging [1,2]. Failing to properly account for uncertainties can, in some cases, lead to erroneous estimations or insecure operating conditions. Therefore, a reliable tool to handle several possible scenarios and combinations of scenarios is crucial to provide a clear understanding of the expected behaviour of the grid. This paper focuses on how to properly account for uncertainties in power flow (PF) analysis.

1.2. State of art

In the literature, uncertainty in PF analysis has been handled mainly by two types of methods: probabilistic and interval-based.

The probabilistic approach relies on solving multiple instances of the PF problem for several (typically randomly generated) possible scenarios, and then aggregate results. The Monte Carlo method is the most common probabilistic approach. The Monte Carlo method is adequate for off-line analysis and is assumed to yield the "correct results", provided that a sufficiently large amount of samples are considered [3]. However, the computational burden of the Monte Carlo method can be unsuitable for practical purposes, real-time analysis and preventive and/or corrective control actions [4]. For an extensive survey of probabilistic PF methods, the interested reader can refer to [3,5].

Interval-based methods rely on using intervals to model the system, according to a *possibility distribution* obtained from experience and historical data. Sentences such as "load between 0.5 and 1 pu" and "generation around 0.9 pu" can be easily translated into intervals. In [6], the interval Newton method is directly applied to a case of PF analysis with 5 buses, assuming small uncertainty in the nodal injected powers. In [1], the authors use *affine* arithmetic to keep track of correlation between inputs, a feature that is absent from traditional interval arithmetic. While interval arithmetic has a low computational burden with respect to probabilistic methods, the major drawback is its tendency to overestimate the intervals of the solution, especially if input parameters are characterised by wide intervals. Wide intervals can make the solution either of little practical interest or useless.

1.3. Contributions

The technique proposed in this paper deals with uncertainty in PF analysis and balances computational burden and accuracy

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of results. With this aim, we utilise a midpoint-radius representation of intervals and separate the solution of the PF problem into a standard PF problem for the midpoint and an interval problem for the radius. The latter is solved through a carefully designed optimisation problem, where the constraints ensure that the solution reflects all the uncertainty of the input, while the objective function helps to prevent overestimation. The concept of linking the numerical results with uncertainty in the input data is borrowed from backward error analysis [7].

The proposed technique enhances the one presented in [8] where the midpoint and radius problems were solved together, thus leading to a higher computational burden and lower accuracy than the technique proposed in this paper.

The proposed method is tested on the IEEE 57 bus test case system, proving to yield results as accurate as the Monte Carlo method. A study of the computational burden of analogous methods is performed in order to show that the proposed method is competitive with state-of-the-art interval-based techniques and much more efficient than the Monte Carlo method.

1.4. Organisation

The remainder of the article is organised as follows. Section 2 reviews probabilistic and interval-based approaches to deal with uncertainty in PF analysis. Section 3 describes the proposed interval method for PF analysis. Section 4 presents a study of the computational complexity of various methods for PF analysis with uncertainty. Section 5 presents a case-study including a comparison in terms of accuracy with the Monte Carlo method. Finally, Section 6 duly draws conclusions and outlines future work.

2. Uncertainty in PF analysis

This section reviews two main approaches used for dealing with uncertainty in PF analysis. These are the probabilistic approach and the interval-based approach.

2.1. Probabilistic approach

The probabilistic approach models uncertainty as random variables with a certain probability distribution. The above relies on statistical data to obtain the probability distribution of the inputs. A *probabilistic PF* model is defined by extending the PF equations to random variables. The equations are solved to obtain the distribution of the unknown variables. The solution method can be numerical, such as Monte Carlo method, or analytic. In the following, we focus exclusively on the Monte Carlo method as it is the most commonly used and is considered to be the most accurate approach [3].

Monte Carlo method. This is a numerical method to approximate the distribution of an unknown random variable. The method relies on the law of large numbers and sampling and consists of the following steps:

1. Create a number of scenarios by taking samples of known random variables.
2. For every scenario, compute a sample of the unknown variables using a deterministic model, e.g., simulation.
3. Aggregate the results into some relevant parameters.

Algorithm 1 illustrates the Monte Carlo method for probabilistic PF analysis. The algorithm computes the sample mean vector of bus voltages $\bar{v}$, given the distribution functions of bus power injections $\boldsymbol{F_S}(\cdot)$. The function rng($\cdot$) returns a random number in the interval [0, 1], and is used for sampling purposes.

Typically, a high number of samples is needed to achieve accurate results. For this reason, the method can become cumbersome if applied to real-world problems involved in the operation of power systems. Common applications are power system planning and reliability analysis [9], or as a validation tool to test other techniques. Several examples can be found of the latter, in which the results from Monte Carlo method are considered the "correct" ones, e.g., [10,11].

Algorithm 1. Monte Carlo method for probabilistic PF analysis.

Input: Number of samples, n_s. Distribution functions of bus power injections, $\boldsymbol{F_S}(\cdot)$. Deterministic PF equations, $\boldsymbol{s} = \boldsymbol{f}(\boldsymbol{v})$.
Output: Sample mean of bus voltages, $\bar{v}$.
1: **for** h in 1, . . ., n_s **do**
2: r = rng()
3: $\boldsymbol{s}^{(h)} = \boldsymbol{F_S}^{-1}(r)$ {Sampling}
4: $\boldsymbol{v}^{(h)} = \boldsymbol{f}^{-1}\left(\boldsymbol{s}^{(h)}\right)$ {Deterministic PF}
5: **end for**
6: $\bar{v} = \frac{1}{n}\sum_h \boldsymbol{v}^{(h)}$ {Sample mean}

2.2. Interval-based approach

The interval-based approach models uncertainty as intervals without a probability distribution. This allows using expert knowledge in the definition of input intervals, in case statistical data is lacking. An *interval PF* model is defined by extending the PF equations to interval variables. The equations are solved in order to compute interval bus voltages. These methods are self-validated, as interval operations respect the property of isotonicity [12].

Interval-based Newton method. This is a method for bounding the zeros of a differentiable function $\boldsymbol{f}(\cdot): \mathbb{R}^n \to \mathbb{R}^n$. Given an initial interval guess $[\boldsymbol{x}]^{(0)}$, the method computes a series $[\boldsymbol{x}]^{(k)}$, k = 1, 2, . . ., such that,

$$\boldsymbol{x} \in [\boldsymbol{x}]^{(0)}, \quad \boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0} \Rightarrow \boldsymbol{x} \in [\boldsymbol{x}]^{(k)} \subset [\boldsymbol{x}]^{(k-1)} \subset \cdots \subset [\boldsymbol{x}]^{(0)}. \tag{1}$$

The method relies on the mean value theorem, which is applied with vectors in the current interval. This leads to a new interval that include all the zeros. However, the new interval might be overlapping the current one. Therefore, both are intersected in order to compute the next interval in the series. Given the current interval $[\boldsymbol{x}]^{(k)}$, the mean value theorem states that,

$$\boldsymbol{f}(\boldsymbol{x}) \in \boldsymbol{f}(\boldsymbol{y}) + \boldsymbol{J_f}|_{\boldsymbol{x} \in [\boldsymbol{x}]^{(k)}}(\boldsymbol{x} - \boldsymbol{y}), \quad \forall \boldsymbol{x}, \boldsymbol{y} \in [\boldsymbol{x}]^{(k)}, \tag{2}$$

where $\boldsymbol{J_f}$ is the Jacobian matrix of $\boldsymbol{f}(\cdot)$. Note that the Jacobian matrix is evaluated on the whole interval $[\boldsymbol{x}]^{(k)}$. Thus, the result is an interval matrix. Enforcing $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0}$, and choosing $\boldsymbol{y}$ as the midpoint of $[\boldsymbol{x}]^{(k)}$ (denoted by $\check{\boldsymbol{x}}^{(k)}$), yields the following expression.

$$\boldsymbol{x} \in \check{\boldsymbol{x}}^{(k)} - \left(\boldsymbol{J_f}|_{\boldsymbol{x} \in [\boldsymbol{x}]^{(k)}}\right)^{-1} \boldsymbol{f}(\check{\boldsymbol{x}}^{(k)}), \tag{3}$$

which leads to the interval Newton iteration,

$$[\boldsymbol{x}]^{(k+1)} = [\boldsymbol{x}]^{(k)} \cap \left(\check{\boldsymbol{x}}^{(k)} - \left(\boldsymbol{J_f}|_{\boldsymbol{x} \in [\boldsymbol{x}]^{(k)}}\right)^{-1} \boldsymbol{f}(\check{\boldsymbol{x}}^{(k)})\right). \tag{4}$$

The interval Newton method is applied to interval PF analysis in the same way as the Newton–Raphson (NR) method is applied to deterministic PF analysis. This idea was introduced in [6].

Note that the interval Newton iteration requires inverting an interval matrix (or, alternatively, solving a system of interval linear equations). Typical factorisation techniques include interval Gaussian elimination and the Krawczyk's method [13]. However, depending on the structure of the matrix, these methods can either not converge, take too long or simply deliver an impractical result [14].

Affine arithmetic method. A novel method for interval PF analysis has been proposed in [1]. The method relies on affine arithmetic,