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Unified formulation of a family of iterative solvers for power systems analysis

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ABSTRACT

This paper illustrates the construction of a new class of iterative solvers for power flow calculations based on the method of Alternating Search Directions. This method is fit to the particular algebraic structure of the power flow problem resulting from the combination of a globally linear set of equations and nonlinear local relations imposed by power conversion devices, such as loads and generators. The choice of the search directions is shown to be crucial for improving the overall robustness of the solver. A noteworthy advantage is that constant search directions yield stationary methods that, in contrast with Newton or Quasi-Newton methods, do not require the evaluation of the Jacobian matrix. Such directions can be elected to enforce the convergence to the high voltage operative solution. The method is explained through an intuitive example illustrating how the proposed generalized formulation is able to include other nonlinear solvers that are classically used for power flow analysis, thus offering a unified view on the topic. Numerical experiments are performed on publicly available benchmarks for large distribution and transmission systems.

1. Introduction

The power flow problem consists in determining the state of a power system in terms of voltage magnitudes and phase angles at each bus, for given load and generation profiles. This can be achieved through the solution of a set of nonlinear power equilibrium equations by means of a numerical iterative method. Considerable research effort has been put into the development of numerical techniques to solve this problem, many of which have come to the point of being considered as “milestones” of power system simulation and are now extensively used by the power industry [1]. Nonetheless, the ever-evolving technological scenario characterizing the power engineering domain demands for a constant improvement of the numerical methods, in order to keep pace with the new standards of robustness, computational speed and reliability required in simulation tools. This idea is what motivates the research work behind this paper.

1.1. Related work

Historically, power flow studies started with Gauss–Seidel (GS) type methods [2,3], Newton–Raphson's methods (NR) [4,5], or fixed point algorithms based on the admittance or impedance matrix, like the Implicit Z bus method (IZB) [6–14]. Despite their flexibility and low memory usage, GS methods have low convergence rates compared to NR methods, who enjoy optimal quadratic convergence but come with an increased computational cost due to the need of assembling and solving the Jacobian system at each iteration. The Implicit Z bus method has a good convergence rate and avoids the problem of reforming a different linear system at each iteration, however it has a less straightforward way of handling voltage control for PV nodes [15–17]. These three classes of solvers are all extensively documented in the specialized literature [18].

A variety of formulations of NR have been developed in order to address the problem of Jacobian update that is particularly critical in large problems for which the solution of the Jacobian system by means of a direct solver becomes computationally expensive. These include Newton–Krylov methods [19,20], Jacobian-free [21], or partial Jacobian update variants [22] which use an approximation to the Jacobian matrix. Among the most popular approaches is the Fast Decoupled Load Flow Method (FDLF) [23], providing an approximation of the Jacobian based on practical properties of the power flow problem. In this way Newton's method is reduced to

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a sequence of decoupled linear problems for the voltage magnitude and phase angle, whose matrices are kept constant throughout the iterations. The theoretical background of this method has been elucidated from a mathematical viewpoint in subsequent works [24,25].

A major drawback of Newton's and Quasi-Newton's methods is the inability to systematically select the operative solution among the multiple possible solutions of the nonlinear set of equation governing the power flow. It is known that convergence behavior of NR is strongly related to the choice of the initial guess solution and that the basins of attractions of the different solutions have fractal boundaries [26]. Traditional iterative solvers may converge to spurious non operative solutions or simply fail to converge in some cases. The operativeness of the solution cannot be assessed if not with an a posteriori stability analysis. This situation is especially critical when the system is close to its voltage stability margin. Different alternatives exist to overcome this difficulty. One is represented by numerical continuation techniques [27,28], that allow to trace power flow solution paths corresponding to different load factors. Other methods are based on truncated Taylor expansions [29–31] or analytical continuation like the more recent Holomorphic Embedding Load Flow Method (HELM) [32], relying on Padé's approximants. In these methods, if the starting solution used for continuation is an operative one, it is possible to guarantee that the path follows a branch of operative solutions up the incipient voltage instability.

While these techniques are able to enforce the convergence to the operative solution and are computationally fast, they have less flexible modeling capabilities. For instance there are reported difficulties in modeling PV nodes in IZB [33] or HELM [34], whereas this is straightforward in NR method. One possibility is to include all control actions, including voltage control and limit enforcement, in an additional loop external to the power flow solution. This inevitably leads to more iterations since for each control iteration a power flow has to be solved. On the other hand, this strategy also reflects more closely the actions of controllers and the way real power systems are operated.

1.2. Contribution of the present work

With the present work we formulate the power flow iterative solver within a new framework. This "family" of solvers is specifically tailored for the algebraic structure of the system of equations arising from the formulation of the power flow problem, and is defined by two free parameters that can be geometrically interpreted as search directions, as is explained later in this paper. The proposed approach gives a unified formulation for a class of power flow iterative methods. Indeed it is shown how some of the classic methods can be obtained from specific choices of the search directions. A particular emphasis is put on the choice of search directions that is capable to enforce the convergence to operative high voltage solutions, while retaining a relatively simple structure of stationary and decoupled solvers, that is, without needing evaluation and factorization the Jacobian matrix at each iteration.

1.3. Organization of the paper

The layout of the paper is organized as follows: the power flow equations are reviewed in Section 2. Here the notation is also set to make the present paper self contained. The development of the new solver is illustrated in detail in Section 3. Examples are presented in Section 4 where performance issues and treatment of voltage controlled nodes are also discussed. Finally, conclusions are drawn in Section 5.

2. Governing equations and notation

In the remainder of this paper the following notation is adopted:

- Regular capital symbols denote vectors while bold capital symbols denote matrices, i.e. $V \in \mathbb{C}^n$ or $\boldsymbol{Y} \in \mathbb{C}^{n\times n}$.
- The superscript * denotes the complex conjugate while the symbol $\odot$ denotes the Hadamard component-wise product of vectors and $\oslash$ denotes the component-wise quotient of vectors.
- $\boldsymbol{Y}V$ denotes the matrix–vector product.
- The nodal admittance matrix including information on both the grid topology and the characteristics of its power delivery devices is represented by $\boldsymbol{Y}$.
- The vectors S, V and I denote vectors whose components are complex power source, voltage and injected currents respectively.

Writing Kirchhoff's current law at any node the following algebraic linear system is obtained:

$$\boldsymbol{Y}V = I_0 + I, \tag{1}$$

where I_0 is the vector containing the constant current. Currents, voltages and powers are nonlinearly related through power balance equations, which can be written in vector form as:

$$S = V \odot I^*. \tag{2}$$

By incorporating Eq. (2) into (1), the following nonlinear system is obtained:

$$\boldsymbol{Y}V = I_0 + S^* \oslash V^*, \tag{3}$$

Eq. (3) is referred to as the injected current form. Multiplying both right and left hand side by V one obtains the power form:

$$V^* \odot [\boldsymbol{Y}V - I_0] = S^* \tag{4}$$

In these formulations the slack node is transformed into equivalent current sources at adjacent buses, and their contribution is accounted for in the vector I_0, while the corresponding complex equation is eliminated from the system, for more details see [35,36]. Therefore, the $\boldsymbol{Y}$ matrix is in general a $n \times n$ complex matrix, while voltages and currents are vectors of $\mathbb{C}^n$, with $n = N_b$ for single-phase systems, or $n = 3N_b$ for three-phase systems when N_b is the number of buses in the network.

3. Proposed methodology

3.1. The method of Alternating Search Directions

Eq. (3) is the combination of linear global problem (1) and nonlinear local constraints (2). In the derivation of the proposed methodology, the first idea is to consider the augmented system formed by Eqs. (1) and (2), instead of the primitive formulation (3), as in [37].

In this framework, a single nonlinear iteration is conceived as the combination of two steps that are obtained by pairing Eqs. (1) and (2) with additional linear relations between voltages and currents, expressing the so-called search directions.

At iteration l, for a given matrix $\boldsymbol{\alpha} \in \mathbb{C}^{n\times n}$ and initial pair $(V, I)^{[l]}$, an intermediate solution (denoted by superscript $l+(1/2)$) is found from the linear system

$$\begin{cases} I^{[l+(1/2)]} - I^{[l]} = \boldsymbol{\alpha}(V^{[l+(1/2)]} - V^{[l]}) \\ \boldsymbol{Y}V^{[l+(1/2)]} = I_0 + I^{[l+(1/2)]} \end{cases} \tag{5}$$