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Computational efficiency improvement of the Universal Line Model by use of rational approximations with real poles

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ABSTRACT

The Universal Line Model (ULM) is widely used for the simulation of electromagnetic transients in electrical power transmission systems. This paper proposes an improvement in the computational efficiency of the ULM by introducing a methodology for rational approximation of smooth frequency domain responses using real poles only through Vector Fitting (VF). The ULM uses a rational model formulation in pole-residue form, as obtained by VF. The models sometimes include several complex poles, leading to complex states that result in slower computations. Transformation into real states is possible but comes at the cost of more complicated expressions for the convolutions. A straightforward methodology is presented in which the complex poles are replaced with real poles based on the non-dominance property of the complex poles in smooth function approximations. This methodology is demonstrated in asymmetrical overhead line configurations, including single-circuit, double-circuit, and parallel lines. The examples show that replacement with real poles leads to negligible loss of accuracy in time domain simulations.

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1. Introduction

The Universal Line Model (ULM) is a frequency-dependent transmission-line model applicable for the electromagnetic transient simulation of overhead lines and underground cables. The model was presented in [1] and developed in the phase domain [2,3]. The ULM is formulated in terms of a rational approximation of the characteristic admittance $\mathbf{Y}_c$ and a multi-delayed rational approximation of the propagation function matrix $\mathbf{H}$. Several improvements have been introduced—e.g., the trace fitting approach in [4]. These rational function-based models are obtained with high accuracy through the Vector Fitting (VF) method [6] permitting recursive implementations of the resulting convolutions [7].

VF was developed in 1999 [8] and has become one of the most popular techniques [9,10]. VF generally delivers rational approximations that include both real and complex poles and residues. In the case of $\mathbf{Y}_c$, the occurrence of complex poles is very rare, unlike rational approximation for $\mathbf{H}$, in which a fewcomplex poles often

occur at high frequencies. The presence of complex poles leads to complex state variables in the time domain convolutions. It is possible to treat the associated states as complex; however, such an approach has been shown to be computationally inefficient [11]. Different techniques have been proposed to handle complex states more efficiently by use of (1) similarity transformation [12]; (2) real arithmetic [11]; and (3) second-, fourth-, and higher-order blocks [13]. All methods consist of special handling of the complex poles when they arise, leading to more complex implementation and some loss of accuracy in time domain simulations.

The aim of this paper is to increase the computational efficiency and simplify the implementation of the ULM by using real poles only. This is achieved by a novel post-process of the VF result. Because the model will have real state variables only, the need for special handling of complex states is avoided.

In this paper, the theory of the ULM is first briefly reviewed. The proposed methodology for obtaining rational approximations of smooth frequency domain responses with real poles using VF is then described. The approach is justified using the concept of pole dominance. Next, the computational advantages of using real poles in the convolutions are determined. Finally, the advantages of the method are demonstrated in asymmetrical overhead line configurations, including single-circuit, double-circuit, and parallel lines.

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2. Universal Line Model (ULM)

Electromagnetic waves propagating along an n-conductor line or cable are governed by Telegrapher's equations (1) and (2), where $\mathbf{V}$ and $\mathbf{I}$ are the voltage and the current vectors of length n. $\mathbf{Z} = \mathbf{R} + j\omega\mathbf{L}$, and $\mathbf{Y} = \mathbf{G} + j\omega\mathbf{C}$ are symmetrical matrices of dimensions $n \times n$:

$$\frac{d^2\mathbf{V}}{dx^2} = \mathbf{ZYV} \tag{1}$$

$$\frac{d^2\mathbf{I}}{dx^2} = \mathbf{YZI}. \tag{2}$$

At ends $x = 0$ and $x = L$ of the line, the following equations relate incident current waves to those reflected from the other end:

$$\mathbf{I}_0 - \mathbf{Y}_c\mathbf{V}_0 = -\mathbf{H}\left[\mathbf{I}_L + \mathbf{Y}_c\mathbf{V}_L\right] \tag{3}$$

$$\mathbf{I}_L - \mathbf{Y}_c\mathbf{V}_L = -\mathbf{H}\left[\mathbf{I}_0 + \mathbf{Y}_c\mathbf{V}_0\right] \tag{4}$$

where

$$\mathbf{Y}_c = \mathbf{Z}^{-1}\sqrt{\mathbf{ZY}} = \sqrt{(\mathbf{YZ})^{-1}}\mathbf{Y} \quad \text{and} \quad \mathbf{H} = e^{-\sqrt{\mathbf{YZ}}l} \tag{5}$$

with $\mathbf{Z}$, $\mathbf{Y}$, $\mathbf{Y}_c$, $\mathbf{H}$ and l being the line series impedance per unit length, shunt admittance per unit length, characteristic admittance matrix, propagation function matrix, and length, respectively.

The term $\mathbf{Y}_c\mathbf{V}_0$ in (3) is defined as a shunt current at $x = 0$:

$$\mathbf{I}_{sh_0} = \mathbf{Y}_c\mathbf{V}_0. \tag{6}$$

Moreover, denoting $\mathbf{I}_H$ in (3) as

$$\mathbf{I}_{H0} = \mathbf{H}\left[\mathbf{I}_L + \mathbf{Y}_c\mathbf{V}_L\right] = \mathbf{HI}_{far_L} \tag{7}$$

where $\mathbf{I}_{far}$ is the reflected current wave from the opposite line end, the voltage/current relationship at the ends of a transmission line is established.

In the application of the ULM for the multi-conductor examples, the trace fitting approach is commonly implemented. This concept minimizes CPU time and memory requirements [4].

In accordance with the trace fitting approach, $\mathbf{Y}_c$ is approximated in the phase domain giving a rational function-based model with a common set of poles [1–4]:

$$\mathbf{Y}_c \cong \sum_{m=1}^{Ny} \frac{\mathbf{G}_m}{s - p_m} + \mathbf{D}, \tag{8}$$

where $\mathbf{D}$ is a constant matrix, $\mathbf{G}_m$ is a matrix of residues, and N_y is the number of poles.

In case of $\mathbf{H}$ using modal decomposition, we can write

$$\mathbf{H} = \mathbf{TH}_m\mathbf{T}^{-1}, \tag{9}$$

where $\mathbf{T}$ is the eigenvector matrix and $\mathbf{H}_m$ is a diagonal matrix of the form

$$\mathbf{H}_m = diag[e^{-\lambda_1 l}, e^{-\lambda_2 l}, \ldots, e^{-\lambda_n l}] \tag{10}$$

with $\lambda = \sqrt{\mathbf{YZ}}$ being the propagation constant of the conductor line [5].

Idempotent decomposition is used for the synthesis of $\mathbf{H}$ in the phase domain as follows [3]:

$$\mathbf{H} = \sum_{i=1}^{N} \Gamma_i e^{-\lambda_i l} \tag{11}$$

where Γ_i is the idempotent square matrix, which results from multiplying the ith column of $\mathbf{T}$ with the ith row of $\mathbf{T}^{-1}$ [1]. Additionally, the exponential factors in (11) can be decomposed as follows [1–4]

$$e^{-\lambda_i l} = e^{-\lambda_i' l} e^{-s\tau_i}. \tag{12}$$

Then (11) can be expressed as

$$\sum_{i=1}^{N} \Gamma_i e^{-\lambda_i' l} e^{-s\tau_i} \tag{13}$$

where τ_i is the time delays. Finally, Eq. (13) is usually expressed as follows [1–4]

$$\mathbf{H} = \sum_{i=1}^{N} \tilde{\mathbf{H}}_i e^{-s\tau_i} \quad \text{with} \quad \tilde{\mathbf{H}}_i = \Gamma_i e^{-\lambda_i' l}. \tag{14}$$

After time delay identification [1,3] for each mode, $\tilde{\mathbf{H}}_i$ is approximated using the trace fitting approach [4] with a common set of poles. Ultimately, the multi-delayed rational function-based model for $\mathbf{H}$ in the phase domain is given by

$$\mathbf{H} \cong \sum_{i=1}^{N} \left(\sum_{k=1}^{Ni} \frac{\mathbf{R}_{i,k}}{s - p_{i,k}} \right) e^{-s\tau_i} \tag{15}$$

where N is the number of propagation modes, N_i is the number of poles for the ith mode, and $\mathbf{R}_{i,k}$ is a matrix of residues.

The rational function-based approximation of both $\mathbf{H}$ and $\mathbf{Y}_c$ allows recursive convolution implementation in the time domain [7].

3. Rational approximation of smooth frequency domain responses

In this section, a straightforward methodology for obtaining rational function-based models with real poles through VF is described.

3.1. Vector Fitting (VF) iteration

VF is an iterative method that aims to fit a rational model in pole-residue form (16) to a given frequency response dataset in the least squares (LS) sense. In the ULM, the frequency responses are the matrix trace of $\mathbf{Y}_c$ and time-compensated eigenvalues of $\mathbf{H}$ [1–4].

VF is based on solving (16) by relocating a set of heuristically calculated initial poles through an iterative procedure. For usage with ULM, the asymptotic value d is nonzero and zero in the case of $\mathbf{Y}_c$ and $\mathbf{H}$, respectively.

$$F(s) \cong \sum_{n=1}^{N} \frac{c_n}{s - p_n} + d. \tag{16}$$

VF first relocates a set of initial poles (p_i) to better positions by an iterative procedure. Then, the residues (c_n, d) are calculated with the given poles taken as known quantities.

In VF, the stability criterion is satisfied by iteratively flipping any pole in the right half-plane into the left half-plane by changing the sign of the real part [8–10].

3.2. Approximation of smooth functions by use of real poles

In the ULM, the frequency domain responses to be fitted ($\mathbf{Y}_c$ and $\mathbf{H}$) are smooth functions of frequency without resonance peaks. Usage of VF leads to a rational model with guaranteed stable poles; however, some complex conjugate poles and residues may result. Below, it will be observed that it is possible to modify the fitting procedure slightly to guarantee a model with only real poles and residues.