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Dynamic phasors modeling for a single phase two stage inverter

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ABSTRACT

Photovoltaic systems, wind turbines and battery storage systems are all commonly present in energy self-sufficiency and independent producers schemes, since they support distributed generation designs. For both cases, single phase inverters provide power conditioning in order to inject micro-sources power to the grid, demanding multiple tasks such as maximum power tracking, DC to AC voltage conversion, electrical signal filtering and network synchronization, among others. The massive integration of such devices demands an assessment on their impact over the low voltage distribution network to which they are connected to. Hence, an accurate and efficient mathematical modeling is required, for both steady and transient state, in order to provide a robust dynamical simulation to support the designing of coordinated protection schemes and operational control algorithms. The analysis also contributes to ensure network stability within the required quality issues. Therefore, this paper extends the dynamic phasors technique that is a widely employed method for modeling oscillatory systems, in order to develop, simulate and analyze the mathematical model of single phase full bridge inverter. A comparison between different types of numeric methods is made, to find the better option depending of the finality of the study. The technique focuses on three frequencies of interest: the network rate, the boost and the inverter stage frequencies. Based on dynamic phasors information, two PI controls are designed to control the DC an AC voltages, and new formulas for calculation of DC and AC powers are presented. Simulation results in Matlab and Simulink demonstrate the effectiveness of the work.

1. Introduction

The increasing installation of photovoltaic systems to the low voltage distribution network via single-phase inverters requires the development of mathematical models that can accurately and efficiently simulate the dynamic variables that interact within such systems. Such models are required to assure stability, power quality and to contribute in the designing of protection schemes, since they are all required in the analysis of novel configurations of distributed generation schemes such as the microgrids [1].

In fact, microgrids as new electrification schemes, are being dominated by elements of power electronics that provide power conditioning of micro sources such as photovoltaic systems, wind turbines, fuel cells, battery storage systems, etc. Developing a dynamic simulation for each element in the system is a more complex task, especially if the transient analysis of the state variables such as voltages and currents is required. Software applications such as the electromagnetic transient program (EMTP) require detailed models whose integration steps fall into the order of microseconds yielding accurate results but still demanding a very high computational cost [2–4].

The modeling technique of dynamic phasors [5] has proven to be a reliable tool to model and simulate switched electronic systems [5–13], electrical machines [14–18], FACTS [19–24] and others. For most of such applications, only dc and fundamental frequency are considered, i.e. only the first order approximation is derived ($k = 0, 1$); other works perform harmonic analysis [25–33], and few others analyze systems with more than two fundamental frequencies, as it might be the case for the network and switching element in power electronics [34,35]. For example in [34], a model of the induction machine including two frequencies, network and switching of the inverter is presented. Such systems are hybrid because there are two types of signals with oscillatory behavior which interact to each other, such as the network voltage (continuous signal) and the inverter voltage (discrete signal).

Inverters are very important in novel energy systems because they deliver the micro-sources power to the network. For example in [36], a model with dynamic phasors of a single-phase inverter

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List of symbols

Dynamic phasors symbols

$x(\tau)$	complex time domain waveform
$X_k(t)$	kth time varying Fourier coefficient
k	set of selected Fourier coefficients
ω_s	fundamental frequency of the system
T	period time of a cycle
j	imaginary operator

Inverter frequencies

f	fundamental frequency
f_c	converter frequency
f_i	inverter frequency

System symbols

L, L_f	inductances of the boost converter and filtering stage
C, C_f	capacitances of the boost converter and filtering stage
V_{dc}	input voltage direct current system
i_L, i_{L_f}	currents of the respective inductances
V_C, V_{C_f}	voltages of the respective capacitances
u	control signal of the boost converter
s_i	control switches of the inverter
a, b	polarity factors
V_d	discontinuous voltage
h	harmonic of interest

that is connected to the grid is developed. However, this model only considers the inverter stage and models only the fundamental frequency so that the dynamics introduced by the switching frequency of the inverter is omitted. It is important to note that the elements of power electronics are more complex, since they require several stages for extracting and conditioning the micro-sources energy and for delivering such power to the network under appropriate requirements.

In this work a single-phase full-bridge inverter is analyzed and modeled through three sections: a boost stage, which is generally used for the design of MPPT algorithms for photovoltaic systems; a DC to AC inverter stage and a filtering stage. Consequently three different frequencies are thus involved, one for each stage, demanding the development, simulation and comparison of a detailed dynamic phasors model for each case. In addition, formulas based on Fourier coefficients are stablished for the calculation of fundamental and oscillating powers, both in the DC side and in the AC side. Finally, based on the dynamic phasor model, two controllers are designed. The first one is to control the voltage of the DC bus and the second one controls the *rms* output voltage of the inverter part.

Our results show that the simulation of dynamic phasors models considering harmonics $k = 0, 1, \ldots, K$ are faster than a detailed model of the inverter [36], and also provide evidence on the oscillatory behavior of other internal signals that have been omitted by previous phasors models that only consider $k = 1$. It is the interest of the authors to include this model and some of their previous work into a simulation library of microgrids that aims to support the development of renewable energy schemes.

The article is distributed as follows: Section 2 describes the overall system; the basis of the modeling technique through dynamic phasors is presented in Section 3. Section 4 describes a model with differential equations and then, by applying the modeling technique of Section 3, a dynamic phasors model is obtained approximating the original model. The results are discussed in Section 5 while Section 6 draws some conclusions.

2. System description

The single-phase full-bridge inverter that is presented consists of three stages: first a boost converter, which can be used for tracking the maximum power of photovoltaic arrays; second, the inverter stage, which creates the behavior of a sine square wave; and third, the filtering step through an LC arrangement.

Notice that the system deploys signals at different frequencies. First, it has a direct current signal V_{DC}, which may come from a renewable energy source such as a photovoltaic array, fuel cell or a battery storage system. There is the switching frequency f_c of the switch u of the boost converter, which can be of the order of kHz. Other frequency represents the inverting stage that is constituted by the s_i switches, which portrait the switching frequency f_i also in the order of kHz and finally the system's main frequency f, whose nature follows the conventional 50/60 Hz of distribution systems.

3. Outline of the dynamic phasors approach

Dynamic phasors modeling technique approximates a signal from its Fourier coefficients (FC) (Eq. (1)), where these FC must be calculated for both the input variables (Eq. (2)) and the state variables (Eq. (3)) and the relationships between them (Eq. (4)) [6].

$$x(\tau) \approx \sum_{-\infty}^{\infty} X_k(t) e^{jk\omega_s \tau} \tag{1}$$

$$X_k(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau) e^{jk\omega_s \tau} d\tau = \langle x \rangle_k(t) \tag{2}$$

$$\left\langle \frac{dx}{dt} \right\rangle_k = \frac{dX_k}{dt} - jk\omega_s X_k \tag{3}$$

$$\langle xy \rangle_k = \sum_{l=-\infty}^{\infty} (X_{k-l} Y_l) \tag{4}$$

where $\omega_s = 2\pi/T$ and $X_k(t)$ is the kth time varying Fourier coefficient in complex form, also called the dynamic phasors; k is the set of selected Fourier coefficients (e.g., $k = 0, 1, \ldots, K$) that provide a good approximation of the original waveform and also gives valuable information that can be used to design control algorithms and stability analysis, among others. j is the imaginary operator.

3.1. Defining powers with dynamic phasors

In this work we highlight an important advantage of dynamic phasors models, in addition to that reported in the literature. They contain useful information according to the selected harmonics. This information can be used for better design of controllers, as well as for deeper analysis on system performance according to their oscillating dynamics and its interaction within and with other systems.

In [37] the different types of switched power systems are discussed. Below, we define the equations based on the Fourier coefficients for compute these powers. The *total power* supplied by the source to the DC bus is defined by Eq. (5), which is composed of two components: *a constant power* and *an oscillating power*. The first one (Eq. (6)) is determined by the product of the direct component of voltage and current. While the oscillating power defined by Eq. (7), is determined by the harmonics considered in the model. The oscillating power must be minimized for better efficiency, since it represents losses in the system.

$$P_{dc}^{total} = P_{dc}^{0} + P_{dc}^{dist} \tag{5}$$

$$P_{dc}^{0} = V_C^0 i_L^0 \tag{6}$$