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Input–output decoupling of discrete-time nonlinear systems by dynamic measurement feedback

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ABSTRACT

The paper addresses the input–output decoupling problem for discrete-time nonlinear systems. The algebraic method based on difference algebra and differential forms is used to solve the problem by measurement feedback, i.e., by feedback depending only on some measured functions of state variables. Constructive necessary and sufficient solvability conditions are given separately for cases when the system is described by the state equations or by the input–output equations. By specifying the measured functions, one recovers the known conditions for the state feedback or obtains the conditions for the output feedback solution.

1. Introduction

In this paper the input–output (i/o) decoupling problem of nonlinear discrete-time systems is studied. It is an important problem, since for decoupled systems one can control the outputs independently. For possible applications, see [20,8,5,2]. In [20,8] the i/o decoupling was used to control induction motors. In [5] the decoupling approach was used to control the model of proton exchange membrane fuel cells and in [2] to control the heating, ventilation and air-conditioning (HVAC) system.

Most contributions that address the i/o decoupling problem use the state feedback (static or dynamic) to get a solution, see for example [22,23,12,6,17]. Since the states are not always available for measurement and constructing an observer is not an easy task, then alternative option is to use the output or measurement feedback. An additional advantage of an output feedback is its applicability to systems described by i/o equations, not realizable in the state space form. In case of output or measurement feedback only some functions of states are used, and therefore the solvability conditions are more restrictive.

Because majority of papers solve the i/o decoupling problem by state feedback, almost all of them, including also those that rely on output or measurement feedback, assume the system to be described by state equations. Up to the authors knowledge, only the papers [19,21] address the case when the system is given by the set of i/o equations. The structure of the feedback in [19] is very different from the standard output feedback, depending on the values of past inputs and outputs and as such is closer to state feedback. In [21], necessary and sufficient solvability conditions by static output feedback are developed. For systems that are described by state equations the static output and measurement feedback solutions are given in [3,10,25,4,11] for continuous-time systems and in [15,21] for discrete-time case, respectively. The more complicated dynamic output or measurement feedback cases have not been much studied. For continuous-time systems, the partial results are given in [3,26] and for discrete-time systems only in [15]. To be more precise, sufficient conditions in [26] were improved and generalized for discrete-time case in [15], where necessary and sufficient conditions were derived.

This paper develops further the results of the conference paper [15]. The necessary and sufficient conditions are found under which the i/o decoupling problem is solvable for discrete-time nonlinear systems by *dynamic measurement feedback*. Two separate cases are addressed: when the system is described by (1) state equations and by (2) i/o equations. First, the results of [15] are improved. Note that the necessary and sufficient conditions from [15] depend on the *existence* of certain differential one-forms and as such are not completely constructive. In this paper the uniquely defined one-forms are found, which describe solvability of the problem. Second, necessary and sufficient solvability conditions are found for systems, described by the set of i/o equations, which generalize the static output feedback solution from [21]. Finally, the results of this paper bind the state, output and measurement feedback solutions. If the measured output is taken equal to the

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state, then the results of this paper recover the known conditions under which the i/o decoupling problem is solvable by the state feedback. If the measured output equals to the controlled output one obtains the solvability conditions by the output feedback.

The paper is organized as follows. Section 2 addresses the case when the system is described by the state equations, whereas Section 3 studies the case when system is described by the set of i/o equations. Finally, the conclusions are made in Section 4.

2. Systems described by state equations

2.1. Preliminaries

Consider the discrete-time nonlinear system, described by the equations

$$x(t+1)=f(x(t),u(t))$$
$$y_*(t)=h_*(x(t))$$
$$y(t)=h(x(t)), \tag{1}$$

where $x(t)\in X\subset\mathbb{R}^n$ is the state, $u(t)\in U\subset\mathbb{R}^m$ is the input, $y_*(t)\subset Y_*\in\mathbb{R}^q$ is the measured output, $y(t)\subset Y\in\mathbb{R}^m$ is the controlled output and X,U,Y_*,Y are open and dense. It is assumed that the functions f, h_* and h are meromorphic. Hereinafter the compact notations are used. Instead of $x(t)$ and $x(t+k)$ $(k\geq 1)$ we use x and $x^{[k]}$, respectively. In some cases the notation $x^{[0]}:=x$ is also used. Similar notations are used for the other variables. We assume additionally that the system (1) is generically submersive, meaning that on an open and dense subset of $X\times U$,

$$\operatorname{rank}\left[\frac{\partial f(\cdot)}{\partial(x,u)}\right]=n. \tag{2}$$

This assumption allows to define below the inversive difference field $\mathcal{K}$, associated to system (1).

Recall the algebraic formalism based on the difference field and differential one-forms, see [1,9] for more information. Let $\mathcal{K}$ be the field of meromorphic functions in a finite number of variables from the set $\mathcal{C}:=\{x,u^{[k]};k\geq 0\}$. The forward-shift operator $\delta:\mathcal{K}\to\mathcal{K}$ is defined by Eq. (1) in the following way: $\delta x=f(x,u)$, $\delta u^{[k]}=u^{[k+1]}$ for $k\geq 0$ and

$$\delta\phi(x,u,\dots,u^{[k]})=\phi\left(f(x,u),u^{[1]},\dots,u^{[k+1]}\right).$$

Under submersivity assumption, the pair $(\mathcal{K},\delta)$, denoted shortly by $\mathcal{K}$, is a difference field [9].

Introduce the set of symbols $\mathrm{d}\mathcal{C}=\{\mathrm{d}x,\mathrm{d}u^{[k]};k\geq 0\}$ and let $\mathcal{E}:=\operatorname{span}_{\mathcal{K}}\{\mathrm{d}\mathcal{C}\}$ be the vector space spanned over $\mathcal{K}$ by a finite number of elements of $\mathrm{d}\mathcal{C}$. The elements of $\mathcal{E}$, i.e.,

$$\omega=\sum_{i=1}^{n}a_i\mathrm{d}x_i+\sum_{k\geq 0}\sum_{j=1}^{m}b_{kj}\mathrm{d}u_j^{[k]}$$

where only finite number of coefficients $a_i,b_{kj}\in\mathcal{K}$ are nonzero, are called one-forms. The operator δ is extended to $\mathcal{E}$:

$$\delta\left(\sum_j a_j\mathrm{d}\varphi_j\right)=\sum_j\delta(a_j)\mathrm{d}(\delta\varphi_j)$$

where $a_j,\varphi_j\in\mathcal{K}$. Consider a one-form $\omega\in\mathcal{E}$ and a subspace $\Theta=\operatorname{span}_{\mathcal{K}}\{\theta_1,\dots,\theta_p\}\subset\mathcal{E}$. By $\mathrm{d}\omega\wedge\omega=0 \bmod \Theta$, one means that $\mathrm{d}\omega\wedge\omega\wedge\theta_1\wedge\cdots\wedge\theta_p=0$.

The relative degree r_i of the ith output component y_i is defined by $r_i:=\min\left\{k\in\mathbb{N}|\mathrm{d}y_i^{[k]}\notin\operatorname{span}_{\mathcal{K}}\{\mathrm{d}x\}\right\}$. If there does not exist such an integer k, then set $r_i:=\infty$.

One says that the system (1) is right-invertible with respect to the controlled output y if there exist $j_i\in\mathbb{N}$ such that

$$\operatorname{rank}_{\mathcal{K}}\frac{\partial\left(h_1(x^{[j_1]}),\dots,h_m(x^{[j_m]})\right)^T}{\partial u}=m, \tag{3}$$

where $h(x)=(h_1(x),\dots,h_m(x))$ [9]. Since right-invertibility is a necessary condition for system to be i/o decoupled by state feedback [17], from now on we assume that the system (1) is right-invertible.

As in [26], we define for each output component y_i a subspace Ω_i of $\mathcal{X}:=\operatorname{span}_{\mathcal{K}}\{\mathrm{d}x\}$ in the following way:

$$\Omega_i=\left\{\omega\in\mathcal{X}|\forall k\in\mathbb{N}:\delta^k\omega\in\operatorname{span}_{\mathcal{K}}\left\{\mathrm{d}x,\mathrm{d}y_i^{[r_i]},\dots,\mathrm{d}y_i^{[r_i+k-1]}\right\}\right\}. \tag{4}$$

The subspaces Ω_i are instrumental in solution of the i/o decoupling problem, since the forward-shifts of the elements of Ω_i do not depend on du. The following lemma gives an algorithm for computation of the subspaces Ω_i.

Lemma 1 (*[14]*). *The subspace Ω_i may be computed as the limit of the sequence Ω_i^k:*

$$\Omega_i^0=\mathcal{X}$$
$$\Omega_i^{k+1}=\left\{\omega\in\Omega_i^k|\delta\omega\in\Omega_i^k+\operatorname{span}_{\mathcal{K}}\left\{\mathrm{d}y_i^{[r_i]}\right\}\right\},\quad k\geq 0. \tag{5}$$

It is immediate from Lemma 1 that $\omega\in\Omega_i$ satisfies the property $\delta\omega\in\Omega_i+\operatorname{span}_{\mathcal{K}}\{\mathrm{d}y_i^{[r_i]}\}$. That is, Ω_i can be understood as the largest subspace of $\mathcal{X}$, which is invariant with respect to forward-shifting, up to adding an element from the subspace $\operatorname{span}_{\mathcal{K}}\{\mathrm{d}y_i^{[r_i]}\}$. The subspace Ω_i can be also viewed as an analogue of maximal $(h_i^{(r_i)},f)$-invariant codistribution of the system $\dot{x}=f(x)+g(x)u$, $y=h(x)$, as defined in [13].

Suppose $\Omega_i=\operatorname{span}_{\mathcal{K}}\{\mathrm{d}\theta_1,\dots,\mathrm{d}\theta_l\}$. Define the forward-shift of subspace Ω_i element-wise: $\Omega_i^{[1]}=\operatorname{span}_{\mathcal{K}}\left\{\mathrm{d}\theta_1^{[1]},\dots,\mathrm{d}\theta_l^{[1]}\right\}$. Let $\Omega_i^{[0]}:=\Omega_i$, and $\Omega_i^{[k]}:=\left(\Omega_i^{[k-1]}\right)^{[1]}$.

One says that the functions $\varphi_i(y_*,\dots,y_*^{[s-1]},u,\dots,u^{[s-1]})$, $i=1,\dots,m$[1], are *linearizable* by the regular dynamic measurement feedback of the form

$$\eta(t+1)=F(\eta(t),y_*(t),v(t))$$
$$u(t)=H(\eta(t),y_*(t),v(t)), \tag{6}$$

where $v(t)\subset V\in\mathbb{R}^m$ is the new input, $\eta(t)\subset\Lambda\in\mathbb{R}^\rho$ is the state of the feedback and V,Λ are open and dense, if in the closed-loop system

$$\mathrm{d}\varphi_i\in\operatorname{span}_{\mathbb{R}}\{\mathrm{d}y_*^{[s-1]},\dots,\mathrm{d}y_*,\mathrm{d}v\}$$

for $i=1,\dots,m$. In the case, when

$$\mathrm{d}\varphi_i\in\operatorname{span}_{\mathbb{R}}\{\mathrm{d}v\}$$

for $i=1,\dots,m$, the functions φ_i are said to be *strictly linearizable*, see more from [16]. In this paper the strict linearization concept is used to represent the solution of the i/o decoupling problem by dynamic measurement feedback. An algorithm and Theorem 3 are given in the Appendix allowing to check whether the functions $\varphi_i(\cdot)$, $i=1,\dots,m$, are strictly linearizable by feedback (6) or not.

By regularity of feedback, it is meant that (6) defines generically the (η,y_*)-dependent one-to-one correspondence between the variables v and u. Feedback (6) is called static if $\rho:=\dim\eta=0$.

[1] In general, one can linearize any number p of functions φ_i, but in this paper we need to linearize exactly m functions.

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