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On Consensus of Nonlinear Multi-Agent Systems with Output Constraints

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Abstract:
This paper proposes a unified approach to analyze and synthesize consensus control laws for nonlinear leaderless multi-agent systems (MASs) subjected to output constraints. First, we employ the input-output feedback linearization method to derive the linearized models of agents. Accordingly, the consensus problem under output constraints for the initial nonlinear MAS is transformed into an equivalent consensus problem under state constraints for the linearized MAS, which is then reformulated as a network of Lur'e systems. Next, a sufficient condition for consensus and the design of consensus controller gain are derived from solutions of a distributed LMI convex problem. Finally, a numerical example is introduced to illustrate the proposed theoretical approach.

Keywords: Multi-agent Systems; Consensus; Output Constraints; Feedback Linearization; Lur'e Networks; distributed LMI.

1. INTRODUCTION

Multi-agent systems (MASs) have gained much attention since there are a lot of practical applications, e.g., power grids, wireless sensor networks, transportation networks, systems biology, etc, can be formulated, analyzed and synthesized under the framework of MASs. One of the most important and intensively investigated issues in MASs (and their applications) is the consensus problem due to its attraction in both theoretical and applied aspects (Olfati-Saber and Murray, 2004; Olfati-Saber et al., 2007; Ren et al., 2007). Recently, several researches start tackling a more advanced problem namely *constrained consensus* analysis and design for MASs. This problem is significant and realistic since MASs in the real world are subjected to physical constraints on their inputs or states. Practical examples include consensus of vehicles with limited speeds, communication range, and working space; smart buildings energy control with temperature and humidity are required in specific intervals, just to name a few.

For linear MASs, a constrained consensus problem was investigated in Nedic et al. (2010) where the states of agents are required to lie in individual closed convex sets and the final consensus state must belong to the non-empty intersection of those sets. Accordingly, a projected consensus algorithm was proposed and then applied to distributed optimization problems. Following this research line, Lin and Ren (2014) extended the result in Nedic et al. (2010) to the context where communication delays exist. In another work, Lee et al. (2011) studied the state increment by utilizing the model predictive control (MPC) method. In fact, using the MPC framework we can also incorporate input or state constraints, however the computational cost could be high. Another direction to deal with input or state constraints is to employ the so-called discarded consensus algorithms (Liu and Chen, 2012; Wang and Uchida, 2013). Nevertheless, a disadvantage of these approaches as well as in Nedic et al. (2010); Lin and Ren (2014) is that the initial states of agents must belong to some sets specified by the constraints, or in other words the consensus is only local. Moreover, only agents with single integrator dynamics were considered in Liu and Chen (2012); Wang and Uchida (2013).

In order to achieve the global or at least semi-global consensus in presence of input or state constraints, some consensus laws were presented in Meng et al. (2012); Su et al. (2013), but they were only applicable for leader-follower MASs. Another way to tackle the input or state constraints to derive global consensus is to reformulate the constrained MAS as a network of Lur'e systems (Takaba, 2014; Zhang et al., 2014). The paper Takaba (2014) considered bounded-constraint inputs and obtained a sufficient condition for global consensus, but the dynamics of agents is limited to single input. In Zhang et al. (2014), constrained consensus problems were investigated where outputs of agents are assumed to be incrementally bounded or passive. Consequently, sufficient conditions for global consensus were derived in the form of LMI convex problems.

On the other hand, the consensus of nonlinear MASs subject to input, state, or output constraints are more challenging and very few results (Atrianfar and Haeri, 2013; Mehrabian and Khorasani, 2015) have been reported so far. A simple second-order nonlinear leader-follower MAS was considered in Atrianfar and Haeri (2013) where the objective is the consensus of agents' velocities under a bounded input constraint. Accordingly, an adaptive consensus protocol was proposed to solve the problem. However, the control input bound depended entirely

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on the number of agents and weights on graph edges, not a predefined value. Moreover, the control law was very complicated. Next, the paper Mehrabian and Khorasani (2015) considered networks of fully actuated leader-follower Euler-Lagrange systems under input and actuator saturation constraints and switching topologies. With special assumptions on the saturation functions, consensus designs and sufficient conditions for consensus are then derived.

This paper tackles the consensus problem for a fairly general class of leaderless single-input single-output (SISO) nonlinear MASs subject to output and output rate constraints, of which no result has been published hitherto, to the best of our knowledge. Consequently, an approach is proposed to design global output-constrained consensus controllers for those nonlinear MASs. First, the input-output feedback linearization method is employed to obtain linearized models of agents. Next, the output-constrained consensus design for the initial nonlinear MAS is transformed into a state-constrained consensus synthesis for the obtained linearized MAS. And finally, the consensus controller' gain is derived from the solutions of a low-dimension convex LMI problem which can be efficiently solved by existing software in a distributed manner.

The paper is organized as follows. In Section 2, we first briefly introduce graph theory and then present the mathematical model of nonlinear MASs. We then employ the input-output feedback linearization method to obtain linearized models of agents in Section 3.1, and propose a consensus design under output constraints based on those linearized models in Section 3.2. Next, Section 4 gives a numerical example to illustrate our proposed method. Lastly, our main results are summarized in section 5.

The following notations and symbols will be used in the paper. $\mathbb{R}$, $\mathbb{R}_-$, and $\mathbb{C}$ stand for the sets of real, non-positive real, and complex number. $\mathrm{Re}(x)$ denotes the real part of a complex number x. Moreover, $\mathbf{1}_n$ and $\mathbf{0}_n$ denote the $n \times 1$ vector with all elements equal to 1 and 0, respectively; and I_n denotes the $n \times n$ identity matrix. Next, $L_f h(x) \triangleq (\partial h(x)/\partial x) f(x)$ represents the notation for Lie derivative, and $\otimes$ stands for the Kronecker product. On the other hand, $\lambda(A)$ and $\lambda_{\min}(A)$ denotes the eigenvalue set and the eigenvalue with smallest, non-zero real part of A, respectively. In addition, $\succ$ and $\succeq$ denote the positive definiteness and positive semi-definiteness of a matrix. Lastly, $\mathcal{K}_\infty$ denotes the class of scalar function $\gamma(x) : \mathbb{R}_+ \to \mathbb{R}_+$ which is continuous, strictly increasing, unbounded, and $\gamma(0) = 0$; and $\mathcal{KL}$ denotes the class of scalar function $\gamma(x,t) : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ such that $\gamma(.,t) \in \mathcal{K}_\infty$ for each t and $\gamma(x,t) \searrow 0$ as $t \to \infty$.

2. PROBLEM FORMULATION

2.1 Graph Theory

In this paper, we consider the scenario where agents' information are exchanged through a bidirectional communication system. Denote $(\mathcal{G}, \mathcal{V}, \mathcal{E})$ the undirected graph representing the information structure in the considered MAS composing of N agents, where each node in $\mathcal{G}$ stands for an agent and each edge in $\mathcal{G}$ represents the interconnection between two agents; $\mathcal{V}$ and $\mathcal{E}$ represent the set of vertices and edges of $\mathcal{G}$, respectively. There is an edge $e_{ij} \in \mathcal{E}$ if agent i receives information from agent j; and $e_{ij} \in \mathcal{E}$ if and only if $e_{ji} \in \mathcal{E}$. The neighboring set of a vertex i is denoted by $\mathcal{N}_i \triangleq \{j : e_{ij} \in \mathcal{E}\}$. Moreover, let a_{ij} be elements of the adjacency matrix $\mathcal{A}$ of $\mathcal{G}$, i.e. $a_{ij} > 0$ if $e_{ij} \in \mathcal{E}$, $a_{ij} = 0$ if $e_{ij} \notin \mathcal{E}$, and $a_{ij} = a_{ji}\ \forall\ i,j = 1,\ldots,N$. The degree of a vertex i is denoted by $\deg_i \triangleq \sum_{j=1}^{N} a_{ij}$, then the degree matrix of $\mathcal{G}$ is denoted by $\mathcal{D} = \mathrm{diag}\{\deg_i\}_{i=1,\ldots,N}$. Consequently, the Laplacian matrix $\mathcal{L}$ associated to $\mathcal{G}$ is defined by $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

2.2 Nonlinear Multi-Agent System Model

Consider a network of N heterogeneous SISO minimum-phase affine nonlinear agents whose models are described as follows,

$$\begin{aligned} \dot{x}_i &= f_i(x_i) + g_i(x_i)u_i, \\ y_i &= h_i(x_i),\ i = 1,\ldots,N, \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}$, and $y_i \in \mathbb{R}$ are the state vector, input, and output of the ith agent, respectively; $f_i, g_i \in \mathbb{R}^{n_i}$ and $h_i \in \mathbb{R}$ are vector-valued and scalar-valued of continuous, differentiable nonlinear functions.

Definition 1. The affine nonlinear agent (1) is said to have relative degree $r_i > 0$ if

$$\begin{aligned} &L_{g_i} L_{f_i}^k h_i(x_i) = 0 \text{ as } k = 0,\ldots,r_i-2, \\ &L_{g_i} L_{f_i}^k h_i(x_i) \neq 0 \text{ as } k = r_i - 1. \end{aligned} \tag{2}$$

Let us denote $(\mathcal{G}, \mathcal{V}, \mathcal{E})$ the graph representing the communication structure in the MAS with vertex set $\mathcal{V}$ and edge set $\mathcal{E}$. The following assumptions will be employed.

A1: All agents have the same relative degree which is equal to 2.
A2: $\mathcal{G}$ is undirected, connected, and fixed.

Note that assumption A1 is used just for the clarity and simplification of results' presentation. Consequently, we investigate a control scenario where each agent collaborates with others to achieve an output consensus while satisfying some specific constraints on their outputs. These constraints take into account physical limitations of agents in real applications, which are represented as follows.

- **Output constraint:**
$$\underline{y}_i \le y_i(t) \le \overline{y}_i\ \ \forall\, i = 1,\ldots,N,\ \forall\, t > 0. \tag{3}$$
- **Output rate constraint:**
$$\delta\underline{y}_i \le \dot{y}_i(t) \le \delta\overline{y}_i\ \ \forall\, i = 1,\ldots,N,\ \forall\, t > 0. \tag{4}$$

Next, the consensus of agents is defined as follows.

Definition 2. A multi-agent system with dynamics of agents described by (1) and information structure represented by $\mathcal{G}$ is said to reach an output consensus if

$$\lim_{t\to\infty} |y_i(t) - y_j(t)| = 0\ \forall\ i,j = 1,\ldots,N. \tag{5}$$

Subsequently, the control design problem is formulated as follows.

- **Design problem (Output-constrained consensus):** For the given nonlinear MASs with dynamics of agents represented by (1) and the information exchange among agents represented by $\mathcal{G}$, find a control strategy to achieve consensus of agents in the sense