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Multi-Agent Coordination with Lagrangian Measurements ⋆

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Abstract: The paper is devoted to the study of a class of optimization problems with convex objective and constraint functions by using extremum seeking methods. Multi-agent systems with single integrator and unicycle dynamics are considered. The purpose is to develop a control algorithm that steers the agents to the set of saddle points of the associated Lagrangian, assuming that only Lagrangian measurements are available. For solving this problem, extremum seeking loops are introduced both for the states of a system and for Lagrangian multipliers. It is shown that, under certain assumptions, the trajectories of the system obtained approximate the trajectories of an auxiliary system representing a saddle point flow. The main result of the paper states the practical uniform asymptotic stability of the set of saddle points. The algorithm proposed is applied to distributed optimization problems.

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1. INTRODUCTION

In this paper, we consider a class of convex optimization problems of the type

$$\begin{aligned} &\inf_x F(x) \\ &\text{s.t. } g_j(x) \le 0,\ j = \overline{1,m}, \end{aligned} \tag{1}$$

where $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$ is the optimization variable of the agents, $F \in C^2 : \mathbb{R}^n \to \mathbb{R}$ is the convex objective (or cost) function, $g_j \in C^2 : \mathbb{R}^n \to \mathbb{R}$ are convex constraint functions, $n, m \in \mathbb{N}$. The purpose is to minimize a function $F(x)$ under certain restrictions on the optimization variables given by $g_j(x)$. Problems (1) arise in different areas of science and describe a variety of problems in mathematics and engineering, as well as economical, electrical, biological models, etc. An important class of problems (1) is represented by optimization tasks for multi-agent systems. For example, consider the situation when each agent has a target point and a set of prescribed constraints on the distances to other agents. In this case, the goal of the multi-agent system is to reach such an optimal configuration where the distances between agents and their target points are minimal, while the constraints are satisfied (see, e.g., Brunner et al. (2012); Dürr et al. (2013b)).

One approach for solving problem (1) is to compute the saddle points of the Lagrangian

$$L(x, \lambda) = F(x) + \sum_{j=1}^{m} \lambda_j g_j(x), \tag{2}$$

which means the minimization of $L(x, \lambda)$ with respect to x and maximization with respect to the so-called Lagrangian multipliers $\lambda = (\lambda_1, \ldots, \lambda_m)^T \in \mathbb{R}^m_+$, where by $\mathbb{R}^m_+$ we denote the set of m-dimensional real vectors with non-negative entries.

Then the problem of finding the solution of (1) is reduced to the stabilization of the saddle points of (2). This issue was considered e.g. in Arrow et al. (1958); Brown and von Neumann (1950); Kose (1956) and others, where a gradient-like vector field involving a projection operator was constructed.

In this paper, we utilize the extremum-seeking techniques (see, e.g., Krstić and Ariyur (2003); Dürr et al. (2013a)) and saddle-points methods (Arrow et al. (1958); Nazemi (2012)) to develop an algorithm ensuring the convergence of trajectories of a multi-agent system to the solution of the convex optimization problem (1), under the assumption that agents can measure only the values of the Lagrangian (2), i. e. the values of $F(x)$, $g_j(x)$ as well as their gradients are unknown. To the best of our knowledge, this problem was not addressed in the literature so far. However, a similar gradient-free saddle points seeking method was proposed in Dürr et al. (2013b) but under the assumption that the values of restriction functions g_j are measured. Analogous methods were developed in Xu and Soh (2013); Ye and Hu (to be published) and applied to distributed control of networked systems, and to energy consumption control. Moreover, extremum seeking with constraints was considered in such works as Coito et al. (2005); DeHaan and Guay (2005); Dougherty and Guay (2014); Mills and Krstic (2013); Poveda and Quijano (2012). In Guay and Dochain (2014), an extremum seeking control algorithm was also proposed for a class of minimax optimization problems which, however, in general does not cover problems (1).

The present study is motivated by several reasons. From the theoretical viewpoint, it provides a novel saddle-point seeking algorithm for solving convex optimization problems with minimal information, e.g. for cases when the values of constraint functions cannot be measured separately. From the practical point of view, the implementation of such a single broadcast

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control signal strategy in networked systems leads to reducing the amount of information shared between agents, and thus might be of interest for security reasons. A further important property of the proposed algorithm is the possibility to use it for distributed optimization problems. Another novel contribution is the extension of the saddle points seeking algorithm from agents with single-integrator dynamics to unicycle dynamics.

The remainder of the paper is organized as follows. Some facts concerning saddle points algorithms and Lie bracket approximations are reviewed in Section II. In Section III, we propose a saddle points seeking strategy for single integrators, using the Lagrangian values only. In Section IV, by using controls with bounded update rates, we extend the algorithm proposed to the convex optimization problem for a multi-agent system with unicycle dynamic, and illustrate its application to distributed optimization problems. The Appendix contains definitions concerning the practical stability.

2. PRELIMINARIES

2.1 Lie bracket approximation

Here we briefly recall some statements from the paper Dürr et al. (2013a). Consider a control-affine system

$$\dot{x} = f_0(t,x) + \sum_{j=1}^{l} f_j(t,x)\sqrt{\omega}u_j(t,\omega t), \tag{3}$$

where $x = (x_1,\dots,x_n)^T \in \mathbb{R}^n$, $u = (u_1,\dots,u_l)^T \in \mathbb{R}^l$, $x(t_0) = x_0 \in \mathbb{R}^n$, and $\omega \in (0,+\infty)$. It was proposed in the paper Dürr et al. (2013a) to use a such called *Lie bracket system* for the analysis of extremum seeking regulators (3) with T-periodic control functions. The Lie bracket system is defined as follows:

$$\dot{\bar{x}} = f_0(t,\bar{x}) + \sum_{i<j}[f_i,f_j](t,\bar{x})v_{ij}(t), \tag{4}$$

where $[f_i,f_j](t,\bar{x}) = \frac{\partial f_j(t,\bar{x})}{\partial \bar{x}} f_i(t,\bar{x}) - \frac{\partial f_i(t,\bar{x})}{\partial \bar{x}} f_j(t,\bar{x})$ denotes the Lie bracket of vector fields $f_i(t,\bar{x})$ and $f_j(t,\bar{x})$, and $v_{ij} = \frac{1}{T}\int_0^T\int_0^\theta u_j(t,\theta)u_i(t,\tau)d\tau d\theta$. Assume that the vector fields of (3) and controls $u_i(t,\omega t)$ satisfy the following assumptions.

A1 $f_i \in C^2 : \mathbb{R}\times\mathbb{R}^n \to \mathbb{R}^n$, $i = \overline{0,l}$.

A2 For every compact set $\mathscr{C} \subset \mathbb{R}^n$, $|f_i(t,x)|$, $\left|\frac{\partial f_i(t,x)}{\partial t}\right|$, $\left|\frac{\partial f_i(t,x)}{\partial x}\right|$, $\left|\frac{\partial^2 f_j(t,x)}{\partial t\partial x}\right|$, $\left|\frac{\partial [f_j,f_k](t,x)}{\partial t}\right|$, and $\left|\frac{\partial [f_j,f_k](t,x)}{\partial x}\right|$ are bounded, for all $x \in \mathscr{C}$, $t \in \mathbb{R}$, $i = \overline{0,l}$, $j = \overline{1,l}$, $k = \overline{j,l}$.

A3 $u_i : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$, $i = \overline{1,l}$, are Lebesgue-measurable, and there exist $L_i, U_i > 0$ such that $|u_i(t_1,\theta) - u_i(t_2,\theta)| \le L_i|t_1 - t_2|$, for all $t_1,t_2 \in \mathbb{R}$, and $\sup_{t,\theta\in\mathbb{R}}|u_i(t,\theta)| \le U_i$.

A4 there exists $T \in (0,\infty)$ such that $u_i(t,\theta+T) = u_i(t,\theta)$, and $\int_0^T u_i(t,\tau)d\tau = 0$, with $T \in (0,\infty)$, for all $t,\theta \in \mathbb{R}, i = \overline{1,l}$.

Let S be a compact set, and $I \subseteq \mathbb{R}^n$ be such that $\bar{I} \cap S$ is non-empty ($\bar{I}$ denotes the closure of I, and $\|\cdot\|$ denotes the Euclidian norm). Then the following results from Dürr et al. (2013a) hold.

Theorem 1. Let Assumptions A1–A4 be satisfied and let $I \subseteq \mathbb{R}^n$ be a set of initial conditions such that system (4) has unique, uniformly bounded solutions. Then for every bounded set $I_1 \subseteq I$, for every $\xi > 0$ and for every $t_1 \in (0,\infty)$, there exists an $\omega_0 \in (0,\infty)$ such that, for all $\omega \in (\omega_0,\infty)$, $t_0 \in \mathbb{R}$, and $x^0 = x(t_0) \in I_1$, there exist unique solutions x and $\bar{x}$ of systems (3) and (4), respectively, through $x(t_0) = \bar{x}(t_0) = x^0$ which satisfy $\|x(t) - \bar{x}(t)\| < \xi$, for $t \in [t_0, t_0+t_1]$.

Theorem 2. Let Assumptions A1–A4 be satisfied and suppose that a compact set $S \subset \mathbb{R}^n$ is locally uniformly asymptotically stable for (4). Then S is locally practically uniformly asymptotically stable for (3).

The practical stability definition can be found in the Appendix. We will denote by $B_\varepsilon(S)$ the ε-neighborhood of a compact set $S \subset \mathbb{R}^n$, $\varepsilon > 0$: $B_\varepsilon(S) = \{x \in \mathbb{R}^n : d(y,S) = \inf_{y\in S}\|x-y\| < \varepsilon\}$.

2.2 Saddle Points

An important role in solving convex optimization problems of type (1) play the saddle points of the associated Lagrangian (2), which are defined as follows.

Definition 1. Let L is given by (2). A point $(x^*,\lambda^*) \in \mathbb{R}^n \times \mathbb{R}^m_+$ is said to be a *saddle point* of L if it satisfies

$$L(x^*,\lambda) \le L(x^*,\lambda^*) \le L(x,\lambda^*), \text{ for all } x \in \mathbb{R}^n, \lambda \in \mathbb{R}^m_+. \tag{5}$$

Denote a saddle point of the Lagrangian L as $q^* = (x^*,\lambda^*)$, and the set of all saddle points as

$$S_L = \{q^* \in \mathbb{R}^n \times \mathbb{R}^m_+ : q^* \text{ is a saddle point of (2)}\}. \tag{6}$$

The following theorems clarify the relation between saddle points and solutions of problem (1), and provide some important properties of saddle points.

Theorem 3. Suppose $q^* \in \mathbb{R}^n \times \mathbb{R}^m_+$ is a saddle point of the Lagrangian (2). Then the corresponding x^* is a solution of (1).

Theorem 4. Let problem (1) be feasible and its optimal value is finite. Furthermore, assume that the Slater condition holds, i. e. there exist $\bar{x}$ such that $g_j(\bar{x}) < 0$, for all $j = \overline{1,m}$. Then there is no duality gap and there exist at least one Lagrangian multiplier.

Theorem 5. (KKT conditions). Let $F, g_j \in C^1 : \mathbb{R}^n \to \mathbb{R}$, $j = \overline{1,m}$, be convex. Then $q^* \in \mathbb{R}^n \times \mathbb{R}^m_+$ is a saddle point of (2) if and only if the following conditions are satisfied for all $j = \overline{1,m}$:

$$\frac{\partial L(x^*,\lambda^*)}{\partial x}^T = \frac{\partial F(x^*)}{\partial x}^T + \sum_{j=1}^m \lambda_j^* \frac{\partial g_j(x^*)}{\partial x}^T = 0,$$
$$\lambda_j^* g_j(x^*) = 0, \quad g_j(x^*) \le 0.$$

Theorems 3–5 can be found, e.g., in Bertsekas (1995); Rockafellar (1970). Another useful result concerns strictly convex target functions.

Theorem 6. Arrow et al. (1958) Consider L defined by (2). Let $F : \mathbb{R}^n \to \mathbb{R}$ be strictly convex, $g_j : \mathbb{R}^n \to \mathbb{R}$, $j = \overline{1,m}$, be convex, and suppose that the set S_L is non-empty. Then the component x^* of every saddle point $(x^*,\lambda^*) \in S_L$ is unique.

Let x^* be a solution of (1). The functions g_j such that $g_j(x^*) = 0$ are said to be *active constraints*. Let $\mathscr{A}(x^*) \subseteq \{1,\dots,m\} = \{j : g_j(x^*) = 0\}$ be the set of indices of active constraints for each $x^* \in S_L$, and $\mathscr{A}^c(x^*) = \{1,\dots,m\} \setminus \mathscr{A} = \{j : g_j(x^*) < 0\}$. Note that, from Theorem 5, $\lambda_j^* = 0$ for all $j \in \mathscr{A}^c(x^*)$.

3. AGENTS WITH SINGLE-INTEGRATOR DYNAMICS

The purpose of this section is to design a saddle points seeking system which will depend on values of the Lagrangian (2) only. Consider single-integrator agents

$$\dot{x} = u, \tag{7}$$

where $x = (x_1,\dots,x_n)^T \in \mathbb{R}^n$ are the states of agents, $u = (u_1,\dots,u_n)^T \in \mathbb{R}^n$ are the controls. We use controls

$$u_i(L,\omega t) = \alpha_{1i}L(x,\lambda)\sqrt{\omega_i}\cos(\omega_i t) + \alpha_{2i}\sqrt{\omega_i}\sin(\omega_i t), \tag{8}$$

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