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Scaled Consensus Tracking Under Constant Time Delay

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Abstract: Over the past decade, various schemes such as consensus, containment, and formation have been proposed for multi-agent systems. A multi-agent systems is said to have reached consensus when all the agents agree on a common value. Recently, however, scaled consensus was introduced as a new notion in the area of multi-agent systems. With this scheme, the agents reach different initial-state-dependent values. In this paper, the problem of scaled consensus tracking under constant time delay is addressed. Firstly, time delay is introduced to the tracking problem of scaled consensus. Then, a proper Lyapunov-Krasovskii functional is employed in order to study the stability of the system. In addition, it is shown that the maximum allowable time delay may be determined via the resultant LMIs. Finally, the findings of this paper are illustrated using a numerical example.

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1. INTRODUCTION

In the past decade, the concept of multi-agent systems has attracted a lot of attention, such that there is a plethora of research being conducted on the subject. In the literature, multi-agent systems are studied in different categories such as consensus, containment, formations, etc.

Consensus is referred to the state in which all the agents agree on a common initial-state-dependent value. Jadbabaie et al. (2003); Olfati-Saber and Murray (2004); Jinhu and Chen (2005); Ren and Beard (2007); Ren (2007); Gao et al. (2014) have considered the consensus control of first-order dynamics. Similarly, Ren (2008); Wenwu et al. (2010); Qian et al. (2014); Zhou and Liao (2014); Dibaji and Ishii (2015); Feng et al. (2015) may be helpful sources on consensus control of second-order dynamics. Researchers interested in consensus of agents with time delay may also refer to the works of Olfati-Saber et al. (2007); Alexandre et al. (2008); Feng and Wang (2008); Blimana and Ferrari-Trecateb (2008); Liu and Liu (2011); Lin et al. (2014); Ya and Tian (2014), and the references therein.

In the present paper, our main focus is the tracking control of multi-agent systems. Hence, it is necessary to introduce some papers on the subject. Avid researchers may refer to Hong et al. (2006) which has discussed tracking control for multi-agent consensus with an active leader and variable topology. Moreover, Porfiri et al. (2007) have addressed simultaneous tracking and formation control for a team of autonomous agents. This paper proposes a two-level consensus approach, where agents reach agreement on the virtual leader state at one level and reach formation about the virtual leader at the other level. Consensus tracking with bounded control effort and directed switching topologies is studied by Ren (2008), which has proposed a consensus tracking algorithm. In order to have a robust consensus tracking, Guerrero et al. (2010) have used a robust optimal control as a tool to design a trajectory tracking control for multi-agent systems. Robust consensus tracking of a class of second-order multi-agent systems has been addressed by Hu (2012). This article has designed continuous distributed consensus protocols to enable global asymptotic consensus tracking. For higher-order multi-agent systems, Wen et al. (2013) have studied the consensus tracking problem with switching directed topologies and occasionally missing control inputs.

In all of the aforementioned articles, agents agree on a common value, which is usually the average of all the initial conditions. However, Roy (2015) has introduced the concept of tracking dynamics on the scaled-consensus manifold, in which agents reach different initial-state-dependent values and track a forcing input. This can be very useful in different areas such as teleoperation, compartmental mass-action systems, closed queuing networks, and water distribution systems, see Roy (2015).

Our paper introduces constant time delay to the tracking problem of scaled consensus and employs a Lyapunov-Krasovskii approach to check the stability of the system. By checking the feasibility of the resultant LMIs, one may find an interval for the allowable delay. To the best of authors' knowledge scaled consensus tracking under time delay has not been addressed elsewhere.

The subsequent parts of this article are organized as follows: section 2 provides the preliminaries. The problem is formulated in section 3, and the main results are

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presented in section 4. A numerical example is provided in section 5. Finally, conclusions are drawn in section 6.

2. PRELIMINARIES

This section provides some basic graph theory notions, which are essential means of analyzing multi-agent systems. Moreover, Schur complement is briefly explained since it is important in the analysis of LMIs.

2.1 Graph Theory

Graph theory has been widely used in order to describe the topology of the networked systems by means of vertices and edges. Let $G(V, E, k)$ denote a weighted graph, in which $V = \{v_1, v_2, \ldots, v_n,\}$ is the finite set of vertices, E is the set of edges, and $W = [k_{ij}] \in \mathbb{R}^{n\times n}$ represents a weighted adjacency matrix. The adjacency matrix of G is defined such that $k_{ij} > 0$ if $(j, i) \in E$ and $k_{ij} = 0$ otherwise. In addition, the Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ associated with graph G is defined as $l_{ii} = \sum_{j=1,j\neq i}^{n} k_{ij}$ and $l_{ij} = -k_{ij}, i \neq j$. Moreover, a graph is called strongly connected if for every pair of vertices there is a path between them.

2.2 Schur Complement

Given matrices A, B, and C, the following holds:

$$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} > 0 \Leftrightarrow C > 0 \ \ \& \ \ A - BC^{-1}B^T > 0$$

where $A - BC^{-1}B^T$ is the Schur complement of the block C, see Fridman (2014).

3. PROBLEM FORMULATION

This section provides the formulation needed to address the mentioned problem. First, tracking problem for scaled consensus is introduced. Afterwards, time delay is added to the problem. In the next section, the problem is solved using a Lyapunov-Krasovskii functional.

3.1 Tracking on the Scaled-consensus Manifold

Agents track a forcing input $f(t)$ in a strongly-connected graph by introducing the following consensus protocol for $i = 1, \ldots, n$:

$$\dot{x}_i(t) = \mathrm{sgn}(\alpha_i) \sum_{j\in U(i)} k_{ij}(\alpha_j x_j(t) - \alpha_i x_i(t)) + \frac{1}{\alpha_i}\dot{f}(t) \quad (1)$$

where $\alpha = [\alpha_1, \ldots, \alpha_n]^T$ and $[K]_{ij} = k_{ij}$ for $i = 1, \ldots, n$, $j \in U(i)$;$[K]_{ii} = -\sum_{j\in U(i)} k_{ij}$ for $i = 1, \ldots, n$; and $[K]_{ij} = 0$ otherwise. Note that $f(t)$ is a continuously-differentiable real, scalar time function, see Roy (2015).

Lemma 1. Roy (2015) The state $x_i(t)$ of agent i converges to the time function $\frac{1}{\alpha_i}(w^T x(0) + f(t))$, where w^T is the left eigenvector of the state matrix, and $x(0)$ is the initial state vector of the network.

3.2 Scaled-consensus Tracking under Constant Time Delay

In (1), all the agents have immediate access to their own states and those of their neighbors'. In this part, we introduce a constant time delay to the tracking problem. In order to formulate the problem, we rewrite (1) as follows:

$$\dot{x}_i(t) = \mathrm{sgn}(\alpha_i) \sum_{j\in U(i)} k_{ij}(\alpha_j x_j(t-\tau) - \alpha_i x_i(t-\tau)) + \frac{1}{\alpha_i}\dot{f}(t)$$

or equivalently as:

$$\begin{aligned} \dot{X}(t) &= [diag(\mathrm{sgn}(\alpha))]K[diag(\alpha)]X(t-\tau) \\ &\quad + [diag(\frac{1}{\alpha})]\dot{f}(t) \end{aligned} \quad (2)$$

$$X(t) = \phi(t) \quad t \in [-\tau, 0] \quad (3)$$

Where τ is the constant time delay, and X is the stack vector of x_i for $i = 1, \ldots, n$.

In order to analyze the tracking of agents, consider a change of variable as $s_i(t) = x_i(t) - \frac{1}{\alpha_i}f(t)$ and rewrite (1) as follows:

$$\dot{s}_i(t) = \mathrm{sgn}(\alpha_i) \sum_{j\in U(i)} k_{ij}(\alpha_j s_j(t) - \alpha_i s_i(t)) \quad (4)$$

It is obvious that $s_i(t)$ converges to $\frac{1}{\alpha_i}(w^T x(0))$; and as a result, $x_i(t)$ converges to $\frac{1}{\alpha_i}(w^T x(0) + f(t))$, see Roy (2015). Considering (4), one may write (2) as follows:

$$\dot{S}(t) = [diag(\mathrm{sgn}(\alpha))]K[diag(\alpha)]S(t-\tau) \quad (5)$$

where $S = [s_1, \ldots, s_n]^T$. It remains to show that (5) is stable. This indicates that (2) is stable according to the introduced change of variables.

4. MAIN RESULTS

A Lyapunov-Krasovskii approach is opted to check the stability of (5). Furthermore, by checking the feasibility of the resultant LMIs, one may determine the allowable time delay.

Inspired by the work of Xu et al. (2013), we introduce the following method in order to analyze tracking on scaled-consensus manifold, and determine the maximum allowed delay by the resultant LMIs.

Theorem 2. Given a scalar $\bar{\tau} > 0$, the multi-agent system in (5) is asymptotically stable for any delay τ satisfying $0 < \tau \leq \bar{\tau}$ if there exist matrices $P > 0$, $Z > 0$, and $Q = Q^T$ such that

$$\begin{bmatrix} P + Z & -Z \\ -Z & \bar{\tau}Q + Z \end{bmatrix} > 0 \quad (6)$$

and

$$\begin{bmatrix} Q - \frac{1}{\bar{\tau}}Z & PA_1 + \frac{1}{\bar{\tau}}Z & 0 \\ A_1^T P + \frac{1}{\bar{\tau}}Z & -Q - \frac{1}{\bar{\tau}}Z & \bar{\tau}A_1^T Z \\ 0 & \bar{\tau}ZA_1 & -\bar{\tau}Z \end{bmatrix} < 0 \quad (7)$$

where $A_1 = [diag(\mathrm{sgn}(\alpha))]K[diag(\alpha)]$.

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