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Robust Distributed Estimation for Localization in Lossy Sensor Networks

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Abstract: In this paper we address the problem of fault resilient estimation for large-scale systems, where the measurements are possibly corrupted due to faults of low-cost sensors. As a toy application, we consider the problem of localization in Sensor Networks (SN). We propose a distributed solution based on a recently developed generalized descent algorithm. To cope with real-world applications, the algorithm we propose is suitable for an asynchronous implementation and is numerically robust to non ideal communications, i.e., packet-losses. Under mild assumptions, theoretical convergence of the algorithm is shown. The algorithm is compared with a recently developed ADMM-based algorithm for robust state estimation.

Keywords: Multi-agent systems, Robust Estimation, Robust Statistics, Asynchronous Broadcast, Distributed Convex Optimization, Sensor Networks, WSN

1. INTRODUCTION

Nowadays, large-scale and distributed cyber-physical systems, consisting of a multitude of sensors and "smart agents" equipped with mild computational, communication and actuation capabilities, permeate our lives. Because of the size of the systems, low-cost sensors are typically used. However the latter are more prone to random failures, and consequently, one important challenge to face is the systematic quantitative monitoring of the system. Indeed, by affecting the collected measurements, these failures eventually compromise the knowledge of the system' state, usually used for management and control. In order to avoid this issue, two strategies can be followed: (i) the development of suitable fault diagnosis algorithms (see Paradis and Han (2007) for a survey on the topic), consisting in detection, isolation and identification of the fault; (ii) the design of fault resilient state estimation procedures which are able to produce accurate outcomes by automatically filtering out the outliers. These two approaches, which may eventually complement each other, become necessary to implement reliable systems. However, the possibly large scale of these systems makes central monitoring strategies difficult and sometimes impossible to implement. Thus, distributed solutions must be addressed.

Fault detection and bad data analysis have been largely studied in the past. A lot of work has been done on the static analysis of faults. The main idea behind static analysis is to process the measurements residuals through suitable hypothesis tests in order to detect the source of the fault. In Chen et al. (2006) a distributed belief propagation approach is proposed for WSN. With specific applications to electrical power systems, in Korres (2011) a distributed bad data analysis and detection procedure is shown, which is based on the normalized residual test. Choi and Xie (2011) propose a reduced model for distributed wide area monitoring and a bad data analysis based on the χ^2-test. A more recent branch of research regards the development of fault diagnosis strategies for general networks of dynamical systems using sensors networks. In Franco et al. (2006) a distributed hypothesis testing method, based on a belief consensus technique to perform fault diagnosis, is presented. Consensus is exploited in Boem et al. (2011) as well, where the authors propose a distributed strategy which is based on the combination of local fault estimators to reach a common agreement on the fault detection. More recently, Boem et al. (2013) propose a method based on Pareto optimization. Finally, in Keliris et al. (2015) the authors present a distributed scheme for the detection of process and sensors faults for a certain class of nonlinear discrete-time systems.

Regarding distributed state estimation, a vast amount of literature can be found. However, historically, state estimation does not deal with the presence of outliers. In order to deal with bad data analysis, the standard approach consists of two iterative steps: first, state estimation is performed; second, hypothesis tests on the measurements residuals are applied as done in Korres (2011); Choi and Xie (2011). If a bad datum is detected, this is deleted from the data-set and state estimation is performed again. Hypothesis test on the new residuals can confirm or belie the detection. In this sense, this approach iteratively combines standard state estimation with static fault detection procedures, to eventually lead to a fault resilient state estimator.
A different approach is followed in Kekatos and Giannakis (2013), where the authors propose an iterative distributed strategy based on the classical ADMM algorithm to simultaneously solve the state estimation and the fault localization in power systems.

In this work we are interested in developing a fault resilient state estimator rather than a fault detection scheme. Conversely to what is done in Kekatos and Giannakis (2013), where the problem is solved using a least square approach with the introduction of an additional variable to take into account the presence of outliers, we exploit ideas coming from robust statistical analysis (Bloomfield and Steiger, 2012; Huber, 2011) to formulate a suitable convex problem. In particular, the choice of a "1−norm"-based cost function, let us automatically filter out potential outliers in the measurements caused by sensors faults. Inspired by the recent result in Todescato et al. (2015), we provide a distributed algorithm to solve the problem. Starting from a synchronous algorithm which assumes perfect and ideal communications among sensor nodes,

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we modify it to deal with communication non idealities. This is an important aspect since, in real-world large-scale systems, ideal synchronous communications are not likely. The algorithm we propose is based on an asynchronous broadcast communication protocol. Numerically, the algorithm is shown to be robust to communication non-idealities. Under additional mild assumptions on the type of communication non-idealities and on the curvature of our prescribed cost function, convergence of the algorithm is theoretically proven.

We apply the proposed algorithm in the framework of sensors networks localization, even if the strategy applies to a more general setup. Because of the well known performance of the ADMM algorithm, we decide to compare the algorithm with the strategy recently proposed in Kekatos and Giannakis (2013). Since neither asynchronous nor robust implementation of the algorithm in Kekatos and Giannakis (2013) is provided, we suggest one. As shown by the numerical simulations, compared to the ADMM, our robust algorithm has the following features: (i) comparable steady state estimation accuracy; (ii) in scenarios of highly connected graphs, the algorithm is characterized by a faster behavior for both the asymptotic and the transient convergence rate; (iii) in general, conversely to the ADMM, the transient evolution of our algorithm is monotonically decreasing.

1.1 Mathematical Preliminaries

In this paper, $\mathcal{G}(\mathcal{V},\mathcal{E})$ denotes a directed graph, where $\mathcal{V} = \{1,\dots,N\}$ is the set of vertices and $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is the set of directed edges. The graph $\mathcal{G}$ is said to be bidirected if $(i,j) \in \mathcal{E}$ implies $(j,i) \in \mathcal{E}$. An undirected path in $\mathcal{G}$ consists of a sequence of vertices $(i_1, i_2, \dots, i_r)$ such that either $(i_j, i_{j+1}) \in \mathcal{E}$ or $(i_{j+1}, i_j) \in \mathcal{E}$ for every $j \in \{1,\dots,r-1\}$. The bidirected graph $\mathcal{G}$ is said to be *connected* if for any pair of vertices (i,j) there exists a undirected path connecting i to j. Given the directed graph $\mathcal{G}$, the set of neighbors of node i, denoted by $\mathcal{N}_i$, is given by $\mathcal{N}_i = \{j \in \mathcal{V} \,|\, (i,j) \in \mathcal{E}\}$. Moreover, $\mathcal{N}_i^+ = \mathcal{N}_i \cup \{i\}$. Given a directed graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ with $|\mathcal{E}| = M$, let the *incidence matrix* $\mathcal{A} \in \mathbb{R}^{M\times N}$ of $\mathcal{G}$ be defined as $\mathcal{A} = [a_{ei}]$, where $a_{ei} = 1, -1, 0$, if edge e is incident on node i and directed away from it, is incident on node i and directed toward it, or is not incident on node i, respectively. Given a vector v, with v^T we denote its transpose and with $diag(v)$ the diagonal matrix where the i-th diagonal element corresponds to the i-th element of the vector v. The symbol I denotes the identity matrix of suitable dimension.

2. PROBLEM FORMULATION

In the following, we consider a *localization-type* problem in Sensor Networks (Mao et al., 2007) where, starting from a set of noisy measurements, the agents' goal is to estimate their absolute positions. We want to develop a distributed strategy where the agents are allowed to exchange information locally, i.e., between neighbors. Moreover, for real-world applications, the algorithm must be robust to communication non idealities, e.g., packet dropouts, while being resilient to faulty measurements due to possible sensors failures.

Consider a set of N agents/sensors $\mathcal{V} = \{1,\dots,N\}$, where each agent is described by a state vector $x_i \in \mathbb{R}^{n_i}$. For our purpose and for ease of notation, we restrict the analysis to the scalar case where $n_i = 1$, $\forall i \in \mathcal{V}$.By exploiting graph theoretical tools, we model the SN by means of a bidirected connected measurement graph $\mathcal{G}(\mathcal{V},\mathcal{E})$.

In the following we introduce the measurement model used and we formally state the problem at hand.

2.1 Measurement Model & Fault Resilient Estimation

Assume that each agent collects a certain number of measurements according to the measurement graph $\mathcal{G}$. More specifically, only two types of measurements can be collected. The first are noisy relative distance measurements with respect to neighboring agents, that is, for each $i \in \mathcal{V}$ and $j \in \mathcal{N}_i$, node i measures

$$b_{ij} = x_i - x_j + w_{ij}\,, \quad w_{ij} \sim \mathcal{N}(0, \sigma_{ij}^2)\ .$$

where σ_{ij}^2 denotes the relative measurement noise variance. The second type of measurements is a noisy absolute measurement of the form

$$b_i = x_i + w_i\,, \quad w_i \sim \mathcal{N}(0, \sigma_i^2)\ .$$

where σ_i^2 is the absolute measurement noise variance. By collecting all the state variables in the vector $\mathbf{x} := [x_1,\dots,x_N]^T$ and by defining the measurement matrix H and the vectors of measurements, $\mathbf{b}$, and noises, $\mathbf{w}$, respectively as

$$H := \begin{bmatrix} I \\ \mathcal{A} \end{bmatrix}, \quad \mathbf{b} := \begin{bmatrix} \{b_i\}_{i\in\mathcal{V}} \\ \{b_{ij}\}_{(i,j)\in\mathcal{E}} \end{bmatrix}, \quad \mathbf{w} := \begin{bmatrix} \{w_i\}_{i\in\mathcal{V}} \\ \{w_{ij}\}_{(i,j)\in\mathcal{E}} \end{bmatrix},$$

the overall measurement model[1] can be rewritten in compact form as

$$\mathbf{b} = H\mathbf{x} + \mathbf{w}\,, \qquad \mathbf{w} \sim \mathcal{N}(0, R)\,, \tag{1}$$

where $R := diag(\{\sigma_i^2\}_{i\in\mathcal{V}}\,, \{\sigma_{ij}^2\}_{(i,j)\in\mathcal{E}})$ denotes the noise variance matrix.

In presence of outliers, however, some of the measurements may be corrupted by an extra term, which has a probability distribution that highly differs from that of the expected gaussian noise. By collecting these outliers in the sparse vector $\mathbf{o}$, the measurement model (1) becomes

$$\mathbf{b} = H\mathbf{x} + \mathbf{w} + \mathbf{o}. \tag{2}$$

Remark 1. (Measurement model). We underline that the more general case of multidimensional positions can be easily derived assuming independent measurements along each dimension. Moreover, all the following analysis seamlessly applies to more general measurement model in which the measurements are linear combinations of the states of neighboring nodes. For instance, this is the case for the state estimation in smart electric grids.

As above mentioned, we are willing to design a distributed state estimation procedure which is fault resilient, that is which is able to produce a reliable estimation by automatically filtering out the outliers. Conversely to classical least squares estimation, where the objective is to minimize the weighted squared norm of the residuals, here we follow an approach which is inspired from robust statistical analysis (Bloomfield and Steiger, 2012; Huber, 2011), i.e., *least absolute estimation.* The main idea is to make use of suitable convex costs which, differently to the classical quadratic costs, are locally quadratic only around the origin while they become linear away from it. Thanks to this, small residuals are weighted quadratically as in the classical least squares. On the contrary, big residuals, which usually identify the presence of sensors faults, are weighted linearly. Consequently, the estimator weights and "trusts" more the measurements corresponding to small

[1] We underline the fact that we do not require all the nodes to collect absolute positioning measurements. However, for absolute positioning we require that at least one agent measures it. Conversely, only relative localization is performed.