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Distributed Adaptive Consensus Protocol with Decaying Gains on Directed Graphs ⋆

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Abstract: In this paper we present a distributed adaptive consensus protocol, that solves the cooperative regulator problem for multi-agent systems with general linear time-invariant dynamics and directed, strongly connected communication graphs. The protocol addresses the problems of recent distributed adaptive consensus protocols with large or unbounded coupling gains. These problems are solved by introducing a novel coupling gain dynamics that allows the coupling gains to synchronize and decay to some estimated value. Unlike the static consensus protocols, which require the knowledge of the smallest real part of the non-zero Laplacian eigenvalues to design the coupling gain, the proposed adaptive consensus protocol does not require any centralized information. It can be therefore implemented on agents in a fully distributed fashion.

Keywords: distributed control, adaptive control, multi-agent systems.

1. INTRODUCTION

In past few decades an increasing demand for the cooperation of multiple interconnected systems initiated a great progress made in the design of distributed controllers for networked multi-agent systems. The inspiration came from the collective animal behaviour such as schooling of fish, flocking of birds, herding of quadrupeds and swarming of insect.

The designs of distributed controllers were motivated by the previously developed theoretical results in the centralized control. When the centralized controller is used for the control of a network of agents, the controller views it as a single complex system, therefore the complexity of the centralized controller increases with the complexity of the network. In most applications the centralized controller cannot observe the full state information due to communication constrains between agents. Moreover, centralized controller might fail when the network topology changes, e.g. an agent or an edge is added or dropped. Therefore the distributed control approach was developed for the control of the multi-agent systems. It handles all drawbacks of the centralized approach and enjoys many advantages, such as robustness, flexibility and scalability.

The basic distributed consensus protocols for formation control in networked multi-agent systems are introduced by Fax and Murray (2004), Olfati-Saber and Murray (2004), Olfati-Saber et al. (2007) and Ren et al. (2007). Various approaches of design of distributed controllers on directed communication graphs are summarized by Zhang et al. (2012). The passivity based design of cooperative controllers is introduces by Arcak (2007). A unified viewpoint on design of consensus regulator on directed graph topologies using the synchronizing region is introduced by Li et al. (2010). The design of distributed controllers and observers using state or output-feedback in continuous and discrete-time is considered by Zhang et al. (2011), Zhang et al. (2012) and Hengster-Movric and Lewis (2013).

The static consensus protocols, e.g. by Li et al. (2010) and Zhang et al. (2011) are very popular in the community because of their well developed and simple controller design. However, because the centralized information (knowledge of the graph topology) is required by each agent by the design, they are not fully distributed.

The recently developed adaptive consensus protocols by Li et al. (2013) and Li et al. (2015) propose a solution to this problem. Since they do not rely on any centralized information, the distributed controllers of agents can be implemented independently without using any global information. Nevertheless the benefit from distributiveness suffers from the high control effort and weak robustness.

In Knotek (2016) we present an adaptive consensus protocol to solve the cooperative regulator problem on undirected graphs. The protocol introduces a novel coupling gain dynamics that forces the coupling gains to synchronize and decay to some estimated value. This solves the above mentioned problems of recent adaptive consensus protocols. In this paper we expand these results to solve the cooperative regulator problem on directed strongly connected communication graphs.

This paper is structured as follows. Section 2 introduces the basic notation and graph preliminaries used throughout the paper. Section 3 states the problems that are being solved by the novel adaptive consensus protocol presented in Section 4. Numerical simulations of the introduced protocol are given in Section 5. Section 6 concludes the paper.

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2. PRELIMINARIES

In this paper the following notations and definitions are used. $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ real matrices. A matrix $M = \text{diag}(v)$ for $v \in \mathbb{R}^n$ denotes $\mathbb{R}^{n\times n}$ diagonal matrix with elements of the vector v on the diagonal. Positive (semi)-definite symmetric matrix is denoted by $M \succ (\succeq)0$.

A directed graph is given by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, \ldots, v_N\}$ is a nonempty finite set of nodes and $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is a set of arcs. An arc is an ordered pair of nodes (v_i, v_j), $v_i \neq v_j$, where v_i is a parent node and v_j is a child node, i.e. the information flows from node v_i to node v_j. A directed path of length N from node v_1 to node v_N is an ordered set of distinct nodes $\{v_1, \ldots, v_N\}$ such that $(v_l, v_{l+1}) \in \mathcal{E}$ for all $l \in [1, N-1]$. A directed graph is strongly connected if there exist a directed path from every node to every other node. In the sequel, we assume the graph $\mathcal{G}$ to be directed, strongly connected and simple, i.e. there are no repeated edges or self-loops $(v_i, v_i) \notin \mathcal{E}, \forall i$.

The adjacency matrix $E = [e_{ij}] \in \mathbb{R}^{N\times N}$ associated with the graph $\mathcal{G}$ is defined by $e_{ij} = 1$ if $(v_j, v_i) \in \mathcal{E}$, otherwise $e_{ij} = 0$. Let the degree matrix $D = [d_{ij}] \in \mathbb{R}^{N\times N}$ be a diagonal matrix given by $d_{ii} = \sum_{j\neq i} e_{ij}$. Then the graph Laplacian matrix is defined by $L = D-E$. Denote $p \in \mathbb{R}^N$ the left eigenvector of L, such that $p^T L = 0$.

3. MOTIVATION

Consider a graph $\mathcal{G}$, that consists of N identical agents with general LTI dynamics

$$\dot{x}_i = Ax_i + Bu_i, \quad i = 1, \ldots, N, \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the state, $u_i \in \mathbb{R}^m$ is the input, and $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are constant matrices. The matrix A is not necessarily stable but the pair of matrices (A, B) is assumed to be stabilizable.

The goal is to design a control law to solve the cooperative regulator problem in the sense of $\lim_{t\to\infty} ||x_j - x_i|| = 0$, $\forall i, j = 1, \ldots, N$ without requiring any centralized information. An adaptive control approach proposes a possible solution to this problem.

There has been several proposed distributed adaptive consensus protocols. The adaptive consensus protocol introduced in Li et al. (2013) solves the cooperative regulator problem on undirected connected graphs. The more recent adaptive consensus protocol by Li et al. (2015) solves the cooperative tracking problem on directed graphs containing a spanning tree with the leader as the root node. The distributed adaptive consensus protocols do not require any global information of a communication graph, therefore they are fully distributed. Nevertheless they introduce also several drawbacks.

Since their coupling gain dynamics contains a quadratic term the coupling gain derivative is a monotonically increasing function and the coupling gain values rise as long as there is some error in states between agents. The farther the initial conditions of the agents, the higher the final values of the coupling gains. The coupling gains might therefore reach higher values than it is needed for the network stability. The coupling gains are decoupled, therefore they end up with different final values and the network gets unbalanced, i.e. the agents react differently to the input signal.

Assuming some noise in state measurements, the coupling gains would permanently rise until they reach some physical bound. Therefore the coupling gains could be just statically set to the boundary value and the adaptive consensus protocol would not be necessary.

To solve the cooperative regulator problem on directed strongly connected graphs and address the above mentioned difficulties, we introduce a novel adaptive control protocol, that allows coupling gains to decay and synchronize.

4. ADAPTIVE CONSENSUS PROTOCOL

Let each agent implement a control input in the form

$$u_i = c_i K \sum_{j=1}^{N} e_{ij}(x_j - x_i), \tag{2}$$

where $c_i \geq 0$ is the time-varying coupling gain associated with an i-th agent and $K \in \mathbb{R}^{m\times n}$ is the feedback gain matrix. The i-th agent dynamics is given by

$$\dot{x}_i = Ax_i + c_i BK \sum_{j=1}^{N} e_{ij}(x_j - x_i). \tag{3}$$

Let each agent implement the coupling gain dynamics

$$\dot{c}_i = \sum_{j=1}^{N} e_{ij}(x_j - x_i)^T \Gamma (x_j - x_i) + \sum_{j=1}^{N} e_{ij}(c_j - c_i) - \ell(c_i - \kappa_i), \tag{4}$$

where $\ell > 0$ is a constant, $\kappa_i \geq 0$ is a constant estimated by the i-th agent and $\Gamma \in \mathbb{R}^{n\times n}$ is the adaptation-gain matrix.

The gain matrices K and Γ are designed by the LQR method. Let $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{m\times m}$ be positive definite symmetric matrices, then

$$K = R^{-1}B^T P, \tag{5}$$

$$\Gamma = PBK, \tag{6}$$

where the positive definite symmetric matrix $P \in \mathbb{R}^{n\times n}$ is the unique positive definite solution of the algebraic Riccati equation (ARE)

$$0 = A^T P + PA + Q - PBR^{-1}B^T P. \tag{7}$$

The introduced adaptive consensus protocol (2, 4) is motivated by Li et al. (2013) and Li et al. (2015), however there are several major differences. The coupling gains are associated with each agent as by Li et al. (2015) and not each interconnection as by Li et al. (2013), but the protocol is more similar to Li et al. (2013). This leads to qualitative changes in the network interactions.

The coupling gain dynamics (4) is not a monotonically increasing function as it was in Li et al. (2013) and Li et al. (2015). It consists of three main terms. The first term on the right-hand side $\sum_j e_{ij}(x_j - x_i)^T \Gamma (x_j - x_i)$ is the non-negative quadratic term motivated by the coupling gain dynamics from Li et al. (2013). Its purpose is to push the coupling gains to higher values until the states get synchronized. The second term on the right-hand side