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A Hybrid Systems Approach for Distributed Nonsmooth Optimization in Asynchronous Multi-Agent Sampled-Data Systems ⋆

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Abstract: We study the problem of robust distributed nonsmooth optimization in a network of sampled data systems with separable response maps but coupled dynamics. Each agent of the network is assumed to have an individual clock and an individual nonsmooth output function, as well as set-valued internal dynamics that are coupled with the internal dynamics of its neighboring agents. In order to achieve robust convergence and stability of the optimal point of the response map of the entire network, we design a distributed deterministic model-free logic-based hybrid controller that globally and robustly synchronizes the clocks of the sampled data systems, while at the same time optimizes the response map of every agent by using only sampled measurements of their individual outputs. We present numerical simulations illustrating the results.

Keywords: Hybrid systems, sampled-data systems, multi-agent systems, adaptive control, optimization.

1. INTRODUCTION

Model-free, or black box optimization of dynamical systems, has an extensive history in the adaptive control literature, e.g., Drapper and Li (1951), Krstic and Wang (2000), Nesić et al. (2013). In this setting, the main goal is to control the input of a system towards an optimal operation point that maximizes its *response map*, i.e., its input-to-output steady state mapping, without making use of the mathematical model of the plant. For multiple-input-single-output (MISO) plants with a sampled-data structure this idea has been addressed in Teel and Popovic (2001), Popović (2004), and Khong et al. (2013), using large sampling-times that allow to decouple the continuous-time dynamics of the plant from the discrete-time dynamics of the controller. On the other hand, black-box optimization and learning in networked and multi-agent systems (MAS) using purely *analogue* controllers and purely *discrete*-time controllers has been considered in Guay et al. (2015), Kutadinata et al. (2012), Poveda and Quijano (2015), Ghods and Krstic (2012), and Poveda et al. (2015), for example. An algorithm for global synchronization of clocks in sampled-data systems with trivial dynamics was considered in Teel and Poveda (2015), while model-based optimal sampled-data control of networks was considered in Kobayashi and Hiraishi (2014). To our knowledge, the problem of black-box nonsmooth optimization in networks of asynchronous sampled-data systems remains unexplored.

In this paper we present a novel hybrid algorithm designed to coordinate, synchronize, and optimize a network of agents, each agent being a sampled-data system with an individual clock and stable, but not necessarily smooth, continuous-time dynamics. Specifically, the continuous-time dynamics associated to each agent are characterized by a well-posed differential inclusion, rendering asymptotically stable a locally bounded set-valued map parametrized by the input signals. These continuous-time dynamics are coupled with the dynamics of the neighboring agents, and each agent aims to maximize its own individual response map, which is assumed to be uncoupled from the actions of its neighbors. The mathematical model of the dynamics and output function of each agent are assumed to be unknown. The algorithm is based on the synchronization dynamics presented in Teel and Poveda (2015) for sampled-data systems with trivial dynamics, and the Lyapunov-method for optimization of static nonsmooth maps considered in Teel (2000). The resulting closed-loop system, which is hybrid by nature, renders the optimal set of the entire network robustly asymptotically stable.

The rest of the paper is organized as follows. Section 2 presents some preliminaries in hybrid dynamical systems aligned with the framework presented in Goebel et al. (2012). Section 3 characterizes the types of networks and sampled-data systems considered in this paper. Section 4 presents our main results for the distributed coordination and optimization problem. Section 5 presents a numerical example, and finally Section 6 ends with some conclusions.

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2. PRELIMINARIES

For any $\rho \in \mathbb{R}_{>0}$, the closed ball of appropriate dimension in the Euclidean norm, of radius ρ, is denoted by $\rho\mathbb{B}$. Given a set $M \subset \mathbb{R}^n$ and a $\rho \in \mathbb{R}_{>0}$ we define $M + \rho\mathbb{B}$ as the union of all sets obtained by taking a closed ball of radius ρ around each point in the set M, and $\text{cl}(M)$ as its closure. A set-valued mapping $M_1 : \mathbb{R}^m \rightrightarrows \mathbb{R}^n$ is outer semicontinuous (OSC) if each convergent sequence $(x_i, y_i) \in \text{graph}(M_1)$, where $\text{graph}(M_1) := \{(x, y) \in \mathbb{R}^m \times \mathbb{R}^n : y \in M_1(x)\}$, with limit (x', y'), is such that $(x', y') \in \text{graph}(M_1)$. The set-valued mapping M_1 is said to be locally bounded (LB) if for any compact set $K \subset \mathbb{R}^m$ there exists $r > 0$ such that $M_1(K) := \bigcup_{x\in K} M_1(x) \subset r\mathbb{B}$. We denote by $\mathbf{1}_n$ and $\mathbf{0}_n$ the vectors in $\mathbb{R}^n$ with all entries equal to 1 and 0, respectively, and by $\mathbb{S} \subset \mathbb{R}^2$ the unitary circle in the plane. Given a locally Lipschitz function $f : \mathbb{R}^n \to \mathbb{R}$, we denote by ∂f the generalized gradient of f, see (Clarke, 1990, Chap. 2).

In this paper we consider hybrid dynamical systems (HDS) aligned with the framework of Goebel et al. (2012), represented by the data $\mathcal{H} := \{C, F, D, G\}$, with state $x \in \mathbb{R}^n$, and described by the equations

$$\dot{x} \in F(x) \qquad x \in C \tag{1a}$$

$$x^+ \in G(x) \qquad x \in D, \tag{1b}$$

where the set-valued mappings $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ and $G : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$, called the flow map and the jump map, respectively, describe the evolution of the system when x belongs to the flow set C or/and the jump set D, respectively. Solutions for the HDS (1) are not necessarily unique, and are defined on hybrid time domains. We say that $E \subset \mathbb{R}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ is a compact hybrid time domain if $E = \cup_{j=0}^{J-1}([t_j, t_{j+1}], j)$ for some finite sequence of times $0 = t_0 \leq t_1 \ldots \leq t_J$. The set E is a hybrid time domain if for all $(T, J) \in E$, $E \cap ([0, T] \times \{0, \ldots, J\})$ is a compact hybrid time domain.

Definition 2.1. A function $x : \text{dom}(x) \mapsto \mathbb{R}^n$ is a hybrid arc if $\text{dom}(x)$ is a hybrid time domain and $x(\cdot, j)$ is locally absolutely continuous for each j. A hybrid arc $x : \text{dom}(x) \mapsto \mathbb{R}^n$ is a solution to $\mathcal{H}$ if $x(0, 0) \in \text{cl}(C) \cup D$, and: 1) For all $j \in \mathbb{N}$ and almost all t such that $(t, j) \in \text{dom}(x)$, $x(t, j) \in C$, and $\dot{x}(t, j) \in F(x(t, j))$; 2) For all $(t, j) \in \text{dom}(x)$ such that $(t, j + 1) \in \text{dom}(x)$, $x(t, j) \in D$, and $x(t, j + 1) \in G(x(t, j))$. ■

Definition 2.2. A hybrid solution is said to be complete if its domain is unbounded. A hybrid solution is said to be pre-complete if its domain is compact or unbounded. A hybrid solution is maximal if there does not exist another solution ψ to $\mathcal{H}$ such that $\text{dom}(x)$ is a proper subset of $\text{dom}(\psi)$ and $x(t, j) = \psi(t, j)$ for all $(t, j) \in \text{dom}(x)$. System $\mathcal{H}$ is said to be (pre) complete from a compact set $K_0 \subset \mathbb{R}^n$ if all maximal solutions x with $x(0, 0) \in K_0$, denoting the set $\mathcal{S}_{\mathcal{H}}(K_0)$, are (pre) complete. ■

Throughout this paper we will make use of different stability notions for HDS, namely, *uniform global asymptotic stability* (UGAS) (Goebel et al., 2012, Def. 3.6), *uniform local asymptotic stability* (ULAS) (Goebel et al., 2012, Def. 7.10), and *semi-global practical asymptotic stability* (SGP-AS) (Goebel et al., 2012, Def. 7.18). These stability notions are similar to the standard stability definitions for continuous-time systems, e.g., Khalil (2002).

In order to guarantee good robustness properties for the closed-loop hybrid system, we will consider HDS of the form (1) with data satisfying the following conditions:

(C1) The sets C and D are closed.
(C2) F is OSC and LB relative to C, $C \subset \text{dom}(F)$, and $F(x)$ is convex for every $x \in C$.
(C3) G is OSC and LB relative to D, and $D \subset \text{dom}(G)$.

Hybrid systems satisfying conditions (C1)-(C3) will be referred to as *well-posed*. Note that purely continuous-time systems and purely discrete-time systems can be seen as HDS of the form (1) with empty jump set and empty flow set, respectively.

3. PROBLEM STATEMENT

We consider a network of N sampled-data systems sharing information via a directed network graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, N\}$ is the set of nodes, also called agents, and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of edges. The edge $(i, j) \in \mathcal{E}$ means that agent i affects agent j. For each node i the set of its neighbors consists of all nodes which have directed edges towards i, and it is denoted by $\mathcal{N}_i = \{j : (j, i) \in \mathcal{E}\}$. We make the following assumption regarding the connectivity of the graph $\mathcal{G}$.

Assumption 3.1. The graph $\mathcal{G}$ is assumed to be time-invariant and strongly connected, that is, for all distinct nodes i and j such that $(i, j) \in \mathcal{E}$, there is a time-invariant path from i to j. ■

For every $i \in \mathcal{V}$, each sampled data system is comprised by three elements: 1) a continuous-time plant with state $\theta_i \in \mathbb{R}^{p_i}$ and scalar output $y_i \in \mathbb{R}$, 2) a controllable input signal $u_i \in \mathbb{R}^{n_i}$, and 3) a resetting clock with state $\tau_i \in \mathbb{R}_{\geq 0}$, which characterizes the times at which the sampler and zero-order hold of each agent samples the output y_i of the plant, and updates its input signal u_i. In absence of any control mechanism each of these clocks evolves according to the uncoupled hybrid dynamics

$$\dot{\tau}_i = \omega \qquad \tau_i \in [0, 1] \tag{2a}$$

$$\tau_i^+ = 0 \qquad \tau_i \in \{1\}, \tag{2b}$$

where $\omega \in \mathbb{R}_{>0}$ is a tunable resetting frequency assumed to be the same for all agents. Note that system (2) is a hybrid system that periodically jumps every ω^{-1} seconds.

In the same way, the continuous-time dynamics associated to each sampled-data system $i \in \mathcal{V}$ are characterized by set-valued dynamics of the form

$$\dot{\theta}_i \in f_i\big(\theta, \alpha_i(\theta, u)\big), \qquad y_i = \varphi_i(\theta), \tag{3}$$

where $\theta = (\theta_1^\top, \ldots, \theta_N^\top)^\top \in \mathbb{R}^p$, $u = (u_1^\top, \ldots, u_N^\top)^\top \in \mathbb{R}^n$, $n := \sum_{i=1}^N n_i$, $p := \sum_{i=1}^N p_i$, $\alpha_i : \mathbb{R}^p \times \mathbb{R}^n \to \mathbb{R}^n$ is a (possibly discontinuous) feedback law pre-designed to stabilize the plant, the mapping $f_i : \mathbb{R}^p \times \mathbb{R}^n \rightrightarrows \mathbb{R}^p$ is set-valued, and $\varphi_i : \mathbb{R}^p \to \mathbb{R}$ is the output function. The mappings $(f_i, \alpha_i, \varphi_i)$ are assumed to be *unknown* for every agent $i \in \mathcal{V}$. We stress that, even though the mappings f_i, α_i, and φ_i seem to depend on the overall states θ and u, the communication structure through the network graph $\mathcal{G}$ should be understood to be implicit in the structure of these mappings, which implies that for every $i \in \mathcal{V}$ these mappings throw away components (θ_j, u_j) for which $j \notin \mathcal{N}_i$.