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Cooperative Output Regulation for Uncertain Nonlinear Multi-Agent Systems with Unknown Control Directions ⋆

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Abstract: This paper concerns the global cooperative output regulation problem for a class of nonlinear multi-agent systems with unknown control directions and unknown exosystem. The communication among agents makes it difficult to implement the Nussbaum gain technique to handle the non-identical unknown control directions. Moreover, the unknown parameter in the exosystem, which cannot be easily handled by the estimators in existing results, makes our problem more intricate. To overcome these challenges, we employ a novel internal model candidate to develop a two-layer control scheme for an adaptive distributed controller design. Theoretic stability analysis is presented and a numerical example is given to show the effectiveness of the proposed controller.

Keywords: Output regulation, multi-agent systems, nonlinear systems, cooperative control, adaptive control.

1. INTRODUCTION

This paper considers a class of nonlinear multi-agent systems described as

$$\begin{aligned} \dot{z}_i &= f_i(z_i, y_i, v, w), \\ \dot{y}_i &= g_i(z_i, y_i, v, w) + b_i(w)u_i, \ i = 1, \dots, N, \end{aligned} \tag{1}$$

where $z_i \in \mathbb{R}^n$ is the state, $y_i \in \mathbb{R}$ is the output, $u_i \in \mathbb{R}$ is the control input, and $w \in \mathbb{W} \subset \mathbb{R}^{n_w}$ represents the uncertainty with $\mathbb{W}$ being a compact set. The control coefficients $b_i := b_i(w)$ are with non-identical unknown signs. These signs are called the control directions, or high-frequency gain signs. It is assumed that functions f_i and g_i are polynomials in their arguments and $f_i(0, 0, 0, w) = g_i(0, 0, 0, w) = 0$. The exogenous signal $v \in \mathbb{R}^{n_v}$ is generated by an exosystem

$$\dot{v} = S(\sigma)v, \tag{2}$$

where $\sigma \in \mathbb{S} \subset \mathbb{R}^{n_\sigma}$ is an unknown parameter with $\mathbb{S}$ being a compact set. It is assumed that for any $\sigma \in \mathbb{S}$, all the eigenvalues of $S(\sigma)$ have zero real parts. The initial condition of the exosystem satisfies $v(0) \in \mathbb{V} \subset \mathbb{R}^{n_v}$. The reference signal y_0 is defined as $y_0(t) = q_0(v(t))$ with q_0 being a polynomial in v. The tracking error is $e_i := y_i - y_0$.

In the past decades, cooperative control for multi-agent systems has received a lot of attentions thanks to its value in practical applications (Olfati-Saber and Murray, 2004; Jadbabaie et al., 2003; Ren, 2007). In practice, it is improbable to know every parameter in the controlled systems precisely. Therefore, considerable existing works have been focused on multi-agent systems with uncertainties (Isidori et al., 2014; Su and Huang, 2013; Zhu and Chen, 2014; Tang et al., 2015). Meanwhile, as an effective tool, cooperative output regulation theory has been utilized to solve robust cooperative control problems. Particularly, via nonlinear regulation theory, a robust output synchronization problem for a class of heterogeneous nonlinear multi-agent systems was addressed in Isidori et al. (2014). By studying cooperative output regulation for a group of nonlinear multi-agent systems with unknown leader, Su and Huang (2015) provides a solution for a leader-following consensus problem. Nonetheless, most of the existing works consider uncertainties in either the system or the exosystem. In this paper, we concentrate on cooperative output regulation with uncertainties in both system (1) and exosystem (2).

Unknown control direction in single systems is usually handled by the Nussbaum gain technique. When it comes to multi-agent systems, it is difficult to adapt the Nussbaum gain technique under distributed control scheme. To overcome this difficulty, Chen et al. (2014) and Ding (2015) have developed new types of Nussbaum gains, respectively, for consensus of some special nonlinear multi-agent systems. Then, Liu (2015) employed the Nussbaum gain proposed in Chen et al. (2014) to solve the cooperative output regulation problem of (1). It is noted that, these new Nussbaum gains have the drawbacks of requiring the unknown control directions to be identical. To remove this

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restriction, we proposed an adaptive distributed controller for output regulation of a class of nonlinear multi-agent systems with non-identical unknown control directions and known exosystem in our recent work Guo et al. (2015). However, the controller designed in Guo et al. (2015) relies on the explicit information of the exosystem, thus is inapplicable in this paper.

When the exosystem contains unknown parameter σ, the cooperative output regulation problem becomes more complicated. Most existing results integrate the internal model principle with some adaptive control method to manage the unknown parameter, for instance, Serrani et al. (2001) and Su and Huang (2013). In our previous work Guo et al. (2014) on single systems, an estimator was developed for σ using the certainty equivalence adaptive control approach. However, under the setting of this work, if an estimator for σ is developed, the estimate error will be extremely difficult to eliminate. To overcome this challenge, we employ the internal model candidate proposed in Xu (2016). Unlike the canonical form of internal models used in Guo et al. (2014) and many other works, the construction of this new internal model candidate does not require the information of σ. Therefore, no additional estimator for σ is needed.

To the best of our knowledge, only a few works have investigated the cooperative output regulation problem with both control directions and exosystem being unknown. In Ding (2015), a leader-following consensus problem for a class of nonlinear output feedback multi-agent systems with identical unknown control directions and unknown leader was considered. A distributed controller was designed using the same new Nussbaum gain for each agent under undirected communication graph. In this work, we consider the global cooperative output regulation for a general class of nonlinear multi-agent systems with non-identical unknown control direction and an unknown exosystem. A two-layer control structure is proposed, so that it is possible to remove some restrictions in Ding (2015), namely, identical unknown control directions and identical new Nussbaum gains. The two-layer control structure is composed of an Upper Layer and a Lower Layer. Upper Layer contains a group of designed nodes one-to-one corresponding to the agent nodes, namely reference nodes. Lower Layer is composed of the agent nodes. A distributed controller based on the novel internal model is developed for the reference nodes in Upper Layer to reproduce the reference signal. Then, in Lower Layer, integrating the Nussbaum gain technique and the output regulation theory, we design an adaptive controller for each agent node to track the output of its associated reference node. Hence, all the agents are capable of tracking the common reference signal regardless of their control directions.

The rest of the paper contains the following sections. In Section 2, the global cooperative output regulation problem is formulated and the internal model design is presented. In Section 3, we design the adaptive distributed controller and analyze the stability of the closed-loop system. Some simulation results are illustrated in Section 4 to show the effectiveness of our proposed controller. Lastly, some conclusive remarks will be drawn in Section 5.

Notation. Throughout this paper, we define $\mathrm{col}(x_1, x_2) := (x_1^\top, x_2^\top)^\top$ for column vectors x_1 and x_2, Υ is the set of smooth functions ranging on the interval $[1, \infty)$ and $\mathbb{D} = \mathbb{V} \times \mathbb{W}$. $\|\cdot\|$ is the Euclidean norm; $\mathbb{R}_+$ denotes the set of nonnegative real numbers. A function is said to be sufficiently smooth if it is C^k for sufficient large integer $k \geq 0$ of particular technique requirements. A function $f : \mathbb{R}^n \mapsto \mathbb{R}_+$ is said to be positive definite if $f(x) > 0$ for $x \neq 0$ and $f(0) = 0$. A function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is of class $\mathcal{K}_\infty$ if it is continuous, positive definite, strictly increasing and unbounded.

2. PROBLEM FORMULATION

The multi-agent system concerned in this paper is composed of agents (1) and exosystem (2). These $(N+1)$ nodes are connected by an undirected graph $\mathcal{G}$. The node set is $\mathcal{V} = \{0, 1, 2, \ldots, N\}$, where 0 denotes the exosystem. The weighted adjacency matrix of $\mathcal{G}$ is denoted by $\mathcal{A} = [a_{ij}]$, $i, j \in \mathcal{V}$, while the neighboring set of the ith node is denoted by $\mathcal{N}_i$.

The global cooperative output regulation problem studied in this work is defined as follows.

For the multi-agent systems composed of (1) and (2), design a distributed controller

$$\begin{aligned} \dot{\nu}_i &= \Lambda_i(\nu_i, \nu_j), \ j \in \mathcal{N}_i, \\ u_i &= \Xi_i(y_i, \nu_i), \ i = 1, \ldots, N, \end{aligned} \tag{3}$$

such that, for any initial condition $(z_i(0), y_i(0), \nu_i(0))$ and any $\sigma \in \mathbb{S}$, $\mathrm{col}(v, w) \in \mathbb{D}$, the trajectory of the closed-loop system composed of (1), (2), and (3) exists for all $t \geq 0$, and the tracking errors e_i approach zero asymptotically.

To solve this cooperative output regulation problem, we need some assumptions.

Assumption 1. In $\mathcal{G}$, node 0 can reach every node i, $i = 1, \ldots, N$.

Assumption 2. For any $w \in \mathbb{W}$, $|b_i(w)| > 0$ and each $\mathrm{sgn}(b_i)$, $i = 1, \ldots, N$, is unknown.

Assumption 3. For $i = 1, \ldots, N$, and any $\sigma \in \mathbb{S}$, $\mathrm{col}(v, w) \in \mathbb{D}$, there exist sufficiently smooth functions $\mathbf{z}_i(\sigma, v, w)$ with $\mathbf{z}_i(0, 0, w) = 0$, such that,

$$\frac{\partial \mathbf{z}_i(\sigma, v, w)}{\partial v} S(\sigma) v = f_i(\mathbf{z}_i(\sigma, v, w), q_0(v), v, w).$$

Remark 2.1. Define matrix $\mathcal{H} = [h_{ij}]$ as $h_{ii} = \sum_{j=1}^N a_{ij}$ and $h_{ij} = -a_{ij}$ for $i \neq j$, $i, j = 1, \ldots, N$. Under Assumption 1, all the eigenvalues of $\mathcal{H}$ have positive real parts. Moreover, as $\mathcal{G}$ is undirected, matrix $\mathcal{H}$ is symmetric. Assumption 2 shows the control directions of (1) are unknown and non-identical. Assumption 3 guarantees the existence of the steady-state states of (1), thus we can construct the following steady-state generator (Huang, 2004),

$$\begin{aligned} \frac{\partial \tau_i(\sigma, v, w)}{\partial v} S(\sigma) v &= \Phi_i \tau_i(\sigma, v, w), \\ \mathbf{u}_i(\sigma, v, w) &= \Psi_i \tau_i(\sigma, v, w), \ i = 1, \ldots, N, \end{aligned} \tag{4}$$

where $\tau_i(\sigma, v, w) = \mathrm{col}(\mathbf{u}_i(\sigma, v, w), \dot{\mathbf{u}}_i(\sigma, v, w), \ldots, \mathbf{u}_i^{(s_i-1)}(\sigma, v, w))$ with s_i being some integers. Matrices Φ_i and Ψ_i are