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Explicit analytical tuning rules for digital PID controllers via the magnitude optimum criterion

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ABSTRACT

Analytical tuning rules for digital PID type–I controllers are presented regardless of the process complexity. This explicit solution allows control engineers 1) to make an accurate examination of the effect of the controller's sampling time to the control loop's performance both in the time and frequency domain 2) to decide when the control has to be I, PI and when the derivative, D, term has to be added or omitted 3) apply this control action to a series of stable benchmark processes regardless of their complexity. The former advantages are considered critical in industry applications, since 1) most of the times the choice of the digital controller's sampling time is based on heuristics and past criteria, 2) there is little a–priori knowledge of the controlled process making the choice of the type of the controller a trial and error exercise 3) model parameters change often depending on the control loop's operating point making in this way, the problem of retuning the controller's parameter a much challenging issue. Basis of the proposed control law is the principle of the PID tuning via the Magnitude Optimum criterion. The final control law involves the controller's sampling time T_s within the explicit solution of the controller's parameters. Finally, the potential of the proposed method is justified by comparing its performance with the conventional PID tuning when controlling the same process. Further investigation regarding the choice of the controller's sampling time T_s is also presented and useful conclusions for control engineers are derived.

1. Introduction

It is by far accepted within the industrial control–automation society that the PID control law offers the simplest and yet most efficient solution to many real–world control problems, [1–8]. In modern control applications, for instance in the field of electrical drives and power electronics [9–12], where controllers are digitally implemented, control engineers still tune the PID parameters based on simple tuning rules, past experience, or heuristics [13,14]. This approach, often leads to poor tuning and unacceptable performance of the control loop in terms of reference tracking and disturbance rejection. Poor tuning is mainly observed in cases where there is little a–priori information regarding the model of the process. A representative example over the industry where poor controller tuning is observed, is the vector control of medium voltage motor drives where the range of switching frequency is often a few hundreds Hz. In this case, the controller's tuning[1] is based often on a simple second order model of the motor and a linear dc gain k_p of the modulation scheme[2]. Since, both motor parameters and the modulator's gain change quite frequently depending on the drive's operating point (change of motor's output frequency), high performance of the drive is not always achieved. Specific parameters in the area of medium voltage drives which are considered to change rather frequently are 1) the affect of the temperature to the rotor time constant [15],[3] 2) variation of the linear dc gain k_p of the pulse width modulator when PWM schemes are followed, [16–19]. In both cases, PI controllers are tuned based on these two parameters. For that reason, many are the cases when poor performance of the drive's control loop is observed, since the aforementioned parameters change frequently while the PI controllers stay tuned with the initial nominal values.

Over the literature, many are the tuning rules that assume the existence of the First Order Lag Plus Dead Time (FOLPDT) model as

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[1] Speed PI controller and current PI controller in the inner loop.
[2] If pulse width modulation techniques are followed.
[3] Fast variation of rotor resistance due to winding temperature.

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the basis for developing a control law: a summary of such tuning rules can be found in [20]. These control laws tune the PID's parameters based on the dc gain k_p of the process, the dominant time constant and its time delay d, while ignoring other dynamics of the process. One of these rules which is often used in the area of many industry applications i.e. electrical drives, is the tuning of the PID controller via the well known Magnitude Optimum criterion [21,13]. The principle of the Magnitude Optimum criterion which was introduced by Sartorius and Oldenbourg, is based on the idea of designing a controller which renders the magnitude of the closed loop frequency response as close as possible to unity, in the widest possible frequency range, $|T(j\omega)| \simeq 1$.

In other words, controller parameters are determined such, so that the robustness of the control loop to disturbances occurring at the output of the process, is maximized. Oldenbourg and Sartorius applied the Magnitude Optimum criterion in type–I systems to processes consisting of stable real poles and since then certain works have been proposed towards the method's improvement, [14,22–26].

In this work, the proposed control law extends the application of the Magnitude Optimum criterion to the design of digital PID controllers. Since modern control applications involve digital controller deployment, this work targets on defining an explicit PID solution

1. which tunes the PID controller's parameters explicitly as a function of all modeled process parameters.
2. that involves the sampling time T_s of the controller. Given this explicit solution, control engineers would be able to apply directly the explicit PID tuning conditions and investigate the affect of the sampling time to the control loop's performance both in the time and frequency domain.
3. The analytical expressions regarding the definitions for the P, I and D gains are straightforward and can be easily integrated within the software of a digitally implemented PID controller.

For clearly and properly presenting the proposed method, in Section 2, the explicit solution presented in [27] is shortly presented in Section 2.1, which serves as a fundamental input to the reader to understand the introduction of the sampling time T_s in the proposed control law. Within the same section, the digital implementation of the PID controller is introduced, the analytical proof of which, is presented in Appendix B. In Sections 3, Sections 4 evaluation results are presented focusing on the detrimental effect the choice of the sampling time can have, when regulating the same process via the analog and digital PID controller respectively. The comparison focuses on the control of benchmark process models which are often met over many industry applications. Finally, goal of this work is to provide both the academic and industry society with a feasible control action which shall be able to deliver reliable results that control engineers can reproduce in–house, before deploying the final control action on a real world prototype application.

2. The proposed PID control law

In this section the conventional, revised and the proposed digital PID control action via the Magnitude Optimum criterion is presented. For the paper's consistency, the conventional and the revised analog control law are briefly presented here, since their complete proof has been thoroughly discussed in [14]. The proof of the proposed digital PID control follows the same line as in [14]. In that a general transfer function of the process model is considered and the explicit solution of the gains is derived based on the plant's parameters and the sampling time T_s. The PID control law is presented in Section 2.2, however the whole proof is analytically presented in Appendix B.

2.1. Analog PID controller design

In this section, a short presentation of the analytic tuning rules for analog PID–type controllers via the Magnitude Optimum criterion is presented. Its detailed proof has been presented in [14] and serves as a fundamental input to the reader to further go through the proposed PID digital control law.

To this end, let the plant transfer function consists of $(n-1)$–poles, m–zeros plus a dead time unit in series. Zeros of the plant may lie both in the left or right imaginary half plane. In that, the plant transfer function is defined by

$$G(s) = \frac{s^m\beta_m + s^{m-1}\beta_{m-1} + \cdots + s^2\beta_2 + s\beta_1 + 1}{s^{n-1}\alpha_{n-1} + \cdots + s^3\alpha_3 + s^2\alpha_2 + s\alpha_1 + 1}e^{-sT_d} \tag{1}$$

where $n-1 > m$. The proposed PID–type controller is given by the flexible form

$$C(s) = \frac{1 + sX + s^2Y}{sT_i(1 + sT_{p_n})} \tag{2}$$

allowing its zeros to become conjugate complex. T_{p_n} stands for the unmodelled controller dynamics coming from the controller's implementation. According to Fig. 1, the closed loop transfer function $T(s)$ is given by

$$T(s) = \frac{k_pC(s)G_p(s)}{1 + k_hk_pC(s)G_p(s)} = \frac{N(s)}{D(s)} = \frac{N(s)}{D_1(s) + k_hN(s)}. \tag{3}$$

Polynomials $N(s)$, $D_1(s)$ are equal to

$$N(s) = k_p(1 + sX + s^2Y)\sum_{i=0}^{m}\left(s^i\beta_i\right), \tag{4}$$

$$D_1(s) = sT_ie^{sT_d}\sum_{j=0}^{n}(s^j\alpha_j) \tag{5}$$

respectively, where $\alpha_0 = \beta_0 = 1$ according to (1). Normalizing $N(s)$, $D_1(s)$ by making the substitution $s' = sc_1$ results in

$$N(s') = k_p(1 + s'x + s'^2y)\sum_{i=0}^{m}(s'^iz_i) \tag{6}$$

$$D_1(s') = s't_ie^{s'd}\sum_{j=0}^{n}\left(s'^jr_j\right) \tag{7}$$

respectively. The corresponding normalized terms involved in the control loop are given by $x = \frac{X}{c_1}$, $y = \frac{Y}{c_1^2}$, $t_i = \frac{T_i}{c_1}$, $d = \frac{T_d}{c_1}$, $r_i = \frac{\alpha_i}{c_1^i}$, $\forall\, i = 1, 2, \ldots n,$, $z_j = \frac{\beta_j}{c_1^j}$, $\forall\, j = 1, 2, \ldots m$. The normalized time

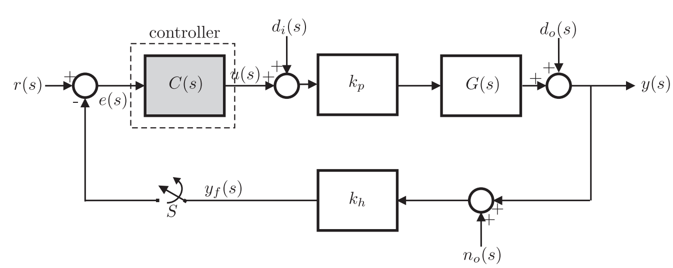

Fig. 1. Block diagram of the closed–loop control system. $G(s)$ is the plant transfer function, $C(s)$ is the controller transfer function, $r(s)$ is the reference signal, $y(s)$ is the output of the control loop, $y_f(s)$ is the output signal after k_h, $d_o(s)$ and $d_i(s)$ are the output and input disturbance signals respectively and $n_o(s)$ is the noise signal process output respectively. k_p stands for the plant's dc gain and k_h is the feedback path. Switch S stands for the border of the open loop transfer function $F_{ol}(s)$ from $r(s)$ to $y_f(s)$.