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Research article

Distributed finite-time tracking for a team of planar flexible spacecraft

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ABSTRACT

The paper presents a distributed finite-time controller for multiple under-actuated spacecraft with flexible appendages to track a virtual leader with stationary states under an undirected communication graph. Each spacecraft of concern is simplified as a free-floating hub-beam system, which is an under-actuated Euler-Lagrange system by nature since only the hub is driven. In the undirected communication graph, it is assumed that only one spacecraft can receive the information from the virtual leader. A distributed finite-time control law is presented for such a multi-agent system. The closed-loop system is proven to converge to the desired states within a finite time via Lyapunov theory and homogeneous method. Finally, a comparison is made between the proposed controller and the PD controller to show the better performance of the proposed controller.

1. Introduction

Significant attention has been paid to the distributed co-operative control for a team of spacecraft because of the advantage of the higher robustness, lower cost, shorter development cycle and more flexibility than a single spacecraft [1,2]. The relevant studies have become quite active over the past decades. For example, Bandyopadhyay et al. reviewed 39 missions of multiple small spacecraft and pointed out that the technology for constellation missions by using small satellites had been matured [1]. Nazari et al. studied the formation of several rigid spacecraft in the framework of geometric mechanics and proposed a decentralized consensus controller with a constant communication time delay [2]. Wang et al. proposed the impulsive controller for the formation tracking of a team of identical oscillators with or without the consideration of input delay [3]. However, the existing studies have mainly focused on the formation or consensus of rigid spacecraft.

It is noteworthy that each spacecraft in a space mission trends towards larger and lighter and is usually equipped with some appendages, such as solar panels and antennas. In other words, the individual spacecraft in the space mission should be regarded to be flexible. The control design will be more challenging due to the influence of the rigid-flexible dynamics [4–7]. The researches on the dynamics and control for a single flexible spacecraft are quite active over the past decades [8,9]. Nevertheless, it is nontrivial to extend the control algorithms from the case of a single flexible spacecraft to the case of a team of flexible spacecraft. Only a few efforts have been made for the design of distributed consensus control of multiple flexible spacecraft. For instance, Du and Li proposed a distributed attitude cooperative controller for the attitude synchronization of a team of flexible spacecraft by using the graph theory and the theory of cascaded systems [10]. Zou and his colleagues designed a controller via the back-stepping technique to solve the attitude consensus for a team of flexible spacecraft without the measurement of both the attitude speeds and the modal coordinates under an undirected communication graph [11]. Du et al. presented a distributed controller for the attitude synchronization of a team of two different kinds of spacecraft, i.e., rigid and flexible ones, in the leader-following formation under the assumption that only several members could receive the information from the leader [12]. In addition, the flexible spacecraft is subject to coupled translational motion and attitude regulation due to the existence of flexible appendages [13]. Chen et al. studied an output consensus controller for the assembly of multiple free-floating flexible spacecraft and a collision avoidance controller to eliminate inter-member collision [13]. Chen et al. also developed an assembly controller, using potential field based method, for four flexible spacecraft under a ring topology [14]. However, all the aforementioned coordination control laws are asymptotically stable within an infinite convergence time.

Recent years have witnessed increasing studies on the so-called finite-time control for particles, spacecraft or Euler-Lagrange systems [15–20] because the finite-time controller can ensure that the closed-loop system reaches the equilibrium

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within a finite time and implies higher robustness of dynamic systems [21]. For instance, Cortés showed that the non-smooth gradient flows could drive a multi-agent system to the same states within a finite time [15]. Zheng and his colleagues proposed a finite-time controller for the consensus of a team of second-order dynamic systems in the absence of the measurement of velocities under the assumption that each member can only obtain the information of the position relative to its neighbors [17]. Hu and Zhang presented a finite-time controller for the tracking problem of a team of spacecraft and showed that the desired trajectory can be tracked within a finite time by using homogeneous theory [22]. Gui and Vukovich designed global finite-time controllers with three types of measurements for attitude tracking of a rigid body described using quaternion [23]. The mechanical and electromechanical systems, described via Lagrange equations, are very common in engineering. Hence, the distributed finite-time control for a team of Euler-Lagrange systems has been studied in [16,24]. Chen and his colleagues studied the finite-time cooperative tracking for a team of Euler-Lagrange systems under a leader-follower structure by using the graph theory and the theory of variable structure control [24]. Zhao et al. presented a distributed finite-time controller to solve the tracking problem of multiple Euler-Lagrange systems in the absence of velocity measurements [16].

All the above studies have focused on the finite-time coordinated control for fully actuated systems. To the best knowledge of authors, however, no study has been made on the design of a distributed finite-time controller for multiple flexible spacecraft, each of which is free-floating and under-actuated. The objective of this study is to present a finite-time controller for a team of flexible spacecraft to track a static trajectory under undirected communication graph. It is assumed that only the states of the rigid hub are measurable and driven directly for each flexible spacecraft. The maneuvering of the hub-beam system is, hence, under-actuated. Compared with the previous studies, the main contribution of this study is the presentation of a distributed finite-time controller for a team of under-actuated Euler-Lagrange systems, each of which is a free-floating flexible spacecraft.

The rest part of the paper is organized as follows. Some fundamentals for notations, graph theory and dynamic equations are introduced in Section 2. In Section 3, a finite-time control law is proposed to drive the team to track the constant trajectory of a virtual leader. Afterwards, a case study is given in Section 4 to illustrate the efficacy of proposed controller. Finally, the conclusions are drawn in Section 5.

2. Problem formulation

2.1. Notations

$\mathbb{R}^{m\times n}$ and $\mathbf{0}_{m\times n}$ represent the set of $m\times n$ real matrices and the $m\times n$ matrix with all entries being zero, respectively. $\boldsymbol{I}_{m\times m}$ and $\mathbf{1}_{m\times n}$ are the identity matrix of $m\times m$ and the matrix of $m\times n$ dimensions with all entries being one. $\boldsymbol{A}>\mathbf{0}$ ($\boldsymbol{A}<\mathbf{0}$) implies that the matrix $\boldsymbol{A}$ is positive (negative) definite. Given a vector $\boldsymbol{x}=[x_1\ \ x_2\ \ ..\ .\ \ x_n]^{\mathrm{T}}$, $\mathrm{sig}(\boldsymbol{x})^{\alpha}$ is defined as $[sgn(x_1)|x_1|^{\alpha},\ ...\ ,\ sgn(x_n)|x_n|^{\alpha}]^{\mathrm{T}}$, where $\mathrm{sgn}(\cdot)$ is the standard sign function and $\alpha\in\mathbb{R}$. The Kronecker product of matrices $\boldsymbol{A}\in\mathbb{R}^{m\times n}$ and $\boldsymbol{B}\in\mathbb{R}^{r\times s}$ is defined as

$$\boldsymbol{A}\otimes\boldsymbol{B}=\begin{bmatrix} a_{11}\boldsymbol{B} & \dots & a_{1n}\boldsymbol{B}\\ \cdots & \cdots & \cdots\\ a_{m1}\boldsymbol{B} & \dots & a_{mn}\boldsymbol{B}\end{bmatrix}_{mr\times ns} \tag{1}$$

For matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ with appropriate dimensions, the Kronecker product $\otimes$ has the following properties [3]

$$(\alpha\boldsymbol{A})\otimes\boldsymbol{B}=\boldsymbol{A}\otimes(\alpha\boldsymbol{B}) \tag{2}$$

$$(\boldsymbol{A}+\boldsymbol{B})\otimes\boldsymbol{C}=\boldsymbol{A}\otimes\boldsymbol{C}+\boldsymbol{B}\otimes\boldsymbol{C} \tag{3}$$

$$(\boldsymbol{A}\otimes\boldsymbol{B})(\boldsymbol{C}\otimes\boldsymbol{D})=(\boldsymbol{AC})\otimes(\boldsymbol{BD}) \tag{4}$$

$$(\boldsymbol{A}\otimes\boldsymbol{B})^{\mathrm{T}}=\boldsymbol{A}^{\mathrm{T}}\otimes\boldsymbol{B}^{\mathrm{T}} \tag{5}$$

$$(\boldsymbol{A}\otimes\boldsymbol{B})^{-1}=\boldsymbol{A}^{-1}\otimes\boldsymbol{B}^{-1} \tag{6}$$

Furthermore, $\int_{\mathbf{0}}^{\boldsymbol{x}} tanh[\mathrm{sig}(\boldsymbol{s})^{\alpha_1}]^{\mathrm{T}}d\boldsymbol{s}$ is defined as $\sum_{i=1}^{n}\int_0^{x_i} tanh[\mathrm{sig}(s_i)^{\alpha_1}]ds_i$ for $\boldsymbol{x}=[x_1\ \ x_2\ \ ..\ .\ \ x_n]^{\mathrm{T}}$, $\boldsymbol{s}=[s_1\ \ s_2\ \ ...\ \ s_n]^{\mathrm{T}}$ and $\alpha_1\in\mathbb{R}$. Hence, the time derivative of $\int_{\mathbf{0}}^{\boldsymbol{x}} tanh[\mathrm{sig}(\boldsymbol{s})^{\alpha_1}]^{\mathrm{T}}d\boldsymbol{s}$ can be expressed as $\dot{\boldsymbol{x}}^{\mathrm{T}}tanh[\mathrm{sig}(\boldsymbol{x})^{\alpha_1}]$.

2.2. Graph theory

The communication topology among the flexible spacecraft in a team can be modelled via graph theory. A graph is denoted as $G=(\mathcal{V},\mathcal{E})$, where $\mathcal{V}=(v_1, v_2, \dots, v_N)$ is a finite set of nodes and $\mathcal{E}=\mathcal{V}\times\mathcal{V}$ is a set of ordered pairs of nodes. An edge (v_i, v_j) means that the node j can obtain the information from the node i. The neighbors of node i are denoted as $N_i=\{v_j|(v_j, v_i)\in\mathcal{E}\}$. The edge (v_i, v_j) is undirected if $(v_i, v_j)\in\mathcal{E}$ and $(v_j, v_i)\in\mathcal{E}$. A graph is undirected if it only consists of undirected edges. The undirected communication graph is said to be connected if there exists a path between each pair of distinct spacecraft. The adjacency matrix of a graph G is represented by $A=[a_{ij}]\in\mathbb{R}^{N\times N}$, where $a_{ij}=1$ if $(v_j, v_i)\in\mathcal{E}$ and otherwise $a_{ij}=0$ holds true. In this study, the graph is assumed to be simple, i.e., there are no self-loops, and no multiple edges between the same pairs of nodes in the graph. Hence, $a_{ii}=0$. The Laplacian matrix of a communication graph is defined as $\boldsymbol{L}=[l_{ij}]$, where $l_{ii}=\sum_{j=1}^{N}a_{ij}$ and $l_{ij}=-a_{ij}$ for $i\neq j$ and $i, j=1, 2,\dots, N$.

Under the leader-follower architecture, the connection between the i-th node and the virtual leader is represented by b_i. Denote $\boldsymbol{B}=\mathrm{diag}([b_1, \dots, b_N])$. If the i-th spacecraft is able to receive information from the leader, then $b_i=1$, otherwise $b_i=0$.

2.3. Dynamic equations

As shown in Fig. 1, each flexible spacecraft in the team consists of a rigid hub and two flexible beams, and can be modeled as a free-floating hub-beam system. The flexible spacecraft is described in the floating frame of reference with only the first mode of the

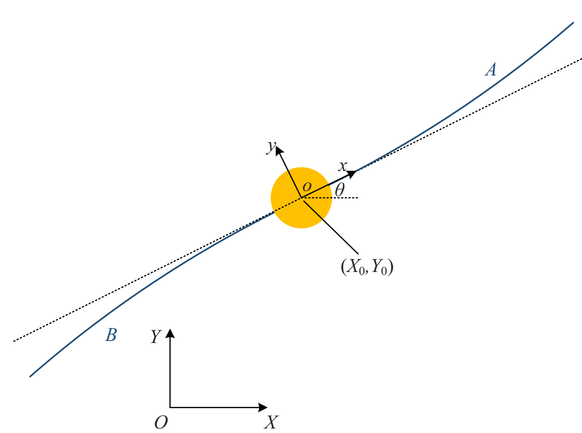

Fig. 1. The schematic of a free-floating spacecraft with two appendages.