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Asymptotic stability and stabilizability of nonlinear systems with delay

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ABSTRACT

This paper is concerned with asymptotic stability and stabilizability of a class of nonlinear dynamical systems with fixed delay in state variable. New sufficient conditions are established in terms of the system parameters such as the eigenvalues of the linear operator, delay parameter, and bounds on the nonlinear parts. Finally, examples are given to testify the effectiveness of the proposed theory.

1. Introduction

Delay naturally occurs in most of the real world problems. So dynamical systems represented by ordinary differential equations with delay seems appropriate to model the real world problems. In particular, applications of delay dynamical systems are in various fields like, biology, economics, robotics, signal processing, control theory [2,7,18,27]. The occurrence of delay in a dynamical system may influence the stability property of the system. For a control system, one can characterize two types of controls, namely open-loop control and closed-loop or feedback control. In practice, feedback control is widely used. A linear time invariant control system is said to be stabilizable if all the unstable states can be made to have stable dynamics by choosing suitable linear feedback control. As time-delay plays an important role in the stability of dynamical system, the stability and stabilizability analysis of delay systems are the important current topics of research in control theory. In the last couple of decades, numerous authors have been working on this topic. In the literature we find two approaches for analyzing the stability property of delay differential systems. (i) Lyapunov like functional and (ii) methods based on the structural properties of the systems.

Trinh and Aldeen [30], Győri et al. [13] and Li and Er [23] presented sufficient conditions for asymptotic stability of linear systems with delayed perturbations using Lyapunov functional. Fridman [11] obtained necessary and sufficient conditions for singularly perturbed linear systems with delay using Lyapunov–Krasovskii functional. In [22], Mori et al. studied stabilizability of linear systems with state delay using linear feedback and Lyapunov functional. The authors in [14,28,10,26,33,16,32,25] established stability conditions for nonlinear systems with delay by using Lyapunov–Krasovskii functionals. Recently in [31], Thuan et al. studied exponential stabilization of time-varying delay systems with nonlinear perturbations using Lyapunov–Krasovskii functionals. Based on Lyapunov–Krasovskii functionals in [20], Liu established sufficient condition for asymptotic stability of interval time-varying delay systems with nonlinear perturbations. The papers [6,15,1,29] and the references therein discuss the stability and stabilization of delay systems with nonlinear perturbations by using the method of Lyapunov functions in the form of Razumikhin.

In [21], Mori obtained several sufficient conditions (delay dependent) for the asymptotic stability of linear time-delay systems. Liu et al. [19], established a new generalized Halanay inequality and using this they studied asymptotic stability of a class of delay differential systems with time-varying structures and delays. In [8], Choi obtained sufficient condition for delay independent global asymptotic stability of a class of delay systems.

In this paper, we focus on asymptotic stability and stabilization of a class of nonlinear systems with time delay in state variable. In general, Lyapunov–Krasovskii stability theory and Razumikhin stability theory are the common approaches to study stability of nonlinear delay systems. Suppose the Lyapunov candidates are simple quadratic functionals, one can easily check negativity of the derivative of Lyapunov candidates using linear matrix inequalities. Constructions of more complicated Lyapunov functionals which give stability conditions are very difficult for systems having

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complicated nonlinear functions [24]. To overcome these difficulties, in this study, we use Gronwall–Bellman Lemma 2.1 and some simple inequalities to obtain sufficient conditions for the delay dependent asymptotic stability and stabilization of some systems. The stability conditions given in this paper are new and improves some results available in literature for certain class of nonlinearity.

The organization of the remainder of the paper is as follows. In Section 2, basic definitions and preliminaries are given. A new sufficient condition for asymptotic stability of the nonlinear delay systems is established in Section 3.1. In Section 3.2, we extend the asymptotic stability result from Section 3.1 to the asymptotic stabilization of a nonlinear control systems with delay in the state variable. Some illustrative examples are presented in Section 4 to validate the proposed theory. Finally, conclusion of the work is given in Section 5.

2. Preliminaries

In this section, basic definitions and preliminaries are given which are useful throughout this paper.

Consider the time-delay system

$$\left.\begin{array}{ll}\dot{x}(t)=Ax(t)+h(t,x(t),x(t-\tau)), & t>0\\ x(t)=\phi(t), & t\in[-\tau,0],\end{array}\right\} \tag{2.1}$$

where $x\in\mathbf{R}^n$ is the state vector, A is a constant $n\times n$ matrix, $h:\mathbf{R}\times\mathbf{R}^n\times\mathbf{R}^n\to\mathbf{R}^n$ is a nonlinear function vector such that $h(t,0,0)=0$, $\tau>0$ is a real constant, ϕ is the continuous vector valued history function and $\|\phi\|=\sup_{t\in[-\tau,0]}\|\phi(t)\|$.

Definition 2.1 ([12]). For the system (2.1), the trivial solution $x(t)=0$ is said to be:

- *stable* if for any $t_0>0$ and for any given $\epsilon>0$ there exists a $\delta>0$ such that $\|\phi\|<\delta$ implies $\|x(t)\|<\epsilon$, $t\ge t_0$.
- *asymptotically stable* if it is stable and if in addition for any $t_0>0$ and any $\epsilon>0$, there exists a $\delta_1>0$ such that $\|\phi\|<\delta_1$ implies $\lim_{t\to\infty}x(t)=0$ and $\|x(t)\|<\epsilon$ for $t\ge t_0$.
- *unstable* if it is not stable.

Note that the stability definition for delay system is same as stability definition for systems without delay, but the only difference is that different assumptions about the initial conditions. Throughout this paper $\|\cdot\|$ denotes the Euclidean norm.

Consider the following linear system without delay:

$$\dot{x}=Ax,\quad x(t_0)=x_0, \tag{2.2}$$

where $x\in\mathbf{R}^n$ is the state vector and A is a nonsingular constant $n\times n$ matrix. Then origin $x=0$ is the equilibrium point of (2.2).

Theorem 2.1 ([3,5]). *The system (2.2) is asymptotically stable if and only if all the eigenvalues of A have negative real parts; (2.2) is unstable for at least one of the eigenvalues of A have positive real part(s); and completely unstable if all the eigenvalues of A have positive real parts.*

Consider the following control system:

$$\dot{x}=Ax+Bu,\quad x(t_0)=x_0 \tag{2.3}$$

where $x\in\mathbf{R}^n$ is the state vector, $u\in\mathbf{R}^m$ is the control, A is a constant $n\times n$ matrix and B is a constant $n\times m$ matrix.

Theorem 2.2 ([17]). *Let S be an arbitrary set of n complex numbers which is symmetric with respect to real axis. Let A and B are the matrices of order $n\times n$ and $n\times m$ respectively. Then there exists a constant matrix K such that the spectrum (set of all eigenvalues) of the matrix $(A+BK)$ is the set S if and only if the rank of the matrix $[B\ AB\ A^2B\ \cdots\ A^{n-1}B]=n$.*

Definition 2.2 ([3,17]). The control system (2.3) is said to be stabilizable if there exists a constant feedback matrix K such that the spectrum of $(A+BK)$ entirely lies in the left-hand side of the complex plane.

Note: From Theorem 2.2, it is clear that the system (2.3) is stabilizable if the rank of $[B\ AB\ A^2B\ \cdots\ A^{n-1}B]=n$.

Lemma 2.1 ([9], *Gronwall–Bellman lemma*). *If*

$$x(t)\le h(t)+\int_{t_0}^{t}k(s)x(s)\mathrm{d}s,\quad t\in[t_0,T)$$

where the functions $h(t)$ and $k(t)$ are nonnegative and continuous on $[t_0,T)$, $T\le+\infty$, then $x(t)$ satisfies

$$x(t)\le h(t)+\int_{t_0}^{t}h(s)k(s)\exp\left[\int_{s}^{t}k(s_1)\mathrm{d}s_1\right]\mathrm{d}s,\quad t\in[t_0,T).$$

3. Main results

3.1. Stability of nonlinear delay systems

Consider the following nonlinear delay system:

$$\left.\begin{array}{ll}\dot{x}(t)=Ax(t)+f(x(t-\tau))+g(x(t)), & t>0\\ x(t)=\phi(t), & t\in[-\tau,0],\end{array}\right\} \tag{3.1}$$

where $x(t)\in\mathbf{R}^n$ is the state vector, A is a constant $n\times n$ matrix, $f,g:\mathbf{R}^n\to\mathbf{R}^n$ are the nonlinear vector functions, $\tau>0$ is a real constant, ϕ is the continuous vector valued initial function and $\|\phi\|=\sup_{t\in[-\tau,0]}\|\phi(t)\|$.

The solution of (3.1) is written as

$$x(t)=e^{At}\phi(0)+\int_0^t e^{A(t-s)}[f(x(s-\tau))+g(x(s))]\mathrm{d}s. \tag{3.2}$$

We assume the following conditions to show the asymptotic stability of system (3.1) with $\|\phi\|<\delta$:

1. The eigenvalues of A has negative real parts.
2. The nonlinear functions $f(x(t-\tau))$ and $g(x(t))$ satisfy the conditions $\lim_{x\to 0}\|g(x)\|/\|x\|=0$ and $\|f(x(t-\tau))\|\le e^{-\lambda\tau}\|x(t-\tau)\|$.
3. $-\lambda+e^{-\lambda\tau}<0$ and $-\lambda+C_1<0$ ($C_1>0$ is a constant given in (3.3)),

where $\lambda=\min\{\lambda_i, i=1,2,\ldots,n\}$, $-\lambda_i$'s ($\lambda_i>0$) are the real parts of the eigenvalues of A.

By the condition 2, there exist a constant $C_1>0$ such that

$$\|g(x(t))\|\le C_1\|x(t)\|\quad\text{when }\|x(t)\|<\delta. \tag{3.3}$$

Using the above first two conditions and (3.3), in the solution (3.2) we get

$$\|x(t)\|\le e^{-\lambda t}\left[|\phi(0)|+e^{-\lambda\tau}\int_0^t e^{\lambda s}\|x(s-\tau)\|\mathrm{d}s+C_1\int_0^t e^{\lambda s}\|x(s)\|\mathrm{d}s\right]. \tag{3.4}$$

When $t\in[0,\tau]$, $x(t-\tau)=\phi(t-\tau)$, hence (3.4) becomes

$$\|x(t)\|\le e^{-\lambda t}\left[|\phi(0)|+e^{-\lambda\tau}\int_0^t e^{\lambda s}\|\phi(s-\tau)\|\mathrm{d}s+C_1\int_0^t e^{\lambda s}\|x(s)\|\mathrm{d}s\right].$$

This implies

$$e^{\lambda t}\|x(t)\|\le\|\phi\|\left(1+e^{-\lambda\tau}\left(\frac{e^{\lambda t}-1}{\lambda}\right)\right)+C_1\int_0^t e^{\lambda s}\|x(s)\|\mathrm{d}s. \tag{3.5}$$

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