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Design of minimum multiplier fractional order differentiator based on lattice wave digital filter

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ABSTRACT

In this paper, a novel design of fractional order differentiator (FOD) based on lattice wave digital filter (LWDF) is proposed which requires minimum number of multiplier for its structural realization. Firstly, the FOD design problem is formulated as an optimization problem using the transfer function of lattice wave digital filter. Then, three optimization algorithms, namely, genetic algorithm (GA), particle swarm optimization (PSO) and cuckoo search algorithm (CSA) are applied to determine the optimal LWDF coefficients. The realization of FOD using LWD structure increases the design accuracy, as only N number of coefficients are to be optimized for Nth order FOD. Finally, two design examples of 3rd and 5th order lattice wave digital fractional order differentiator (LWDFOD) are demonstrated to justify the design accuracy. The performance analysis of the proposed design is carried out based on magnitude response, absolute magnitude error (dB), root mean square (RMS) magnitude error, arithmetic complexity, convergence profile and computation time. Simulation results are attained to show the comparison of the proposed LWDFOD with the published works and it is observed that an improvement of 29% is obtained in the proposed design. The proposed LWDFOD approximates the ideal FOD and surpasses the existing ones reasonably well in mid and high frequency range, thereby making the proposed LWDFOD a promising technique for the design of digital FODs.

1. Introduction

Fractional calculus is an emerging field of research and gained popularity over the integer order system due to its flexibility and performance improvement. In recent years, the fractional order derivative has acquired great attention of research community almost in every field of engineering and science to model various problems of automatic control, image processing, signal processing, fluid dynamics, electromagnetic theory, physics, biology, hydrology and electrical circuits [1–4]. Fractional derivative is used to compute the time derivative of the applied signal. In order to obtain flexibility in design and better design accuracy, the integer order derivative is generalized to fractional order derivative. The integer order derivative $D^n f(x) = \frac{d^n f(x)}{dx^n}$ has been generalized to fractional order derivative $D^\nu f(x) = \frac{d^\nu f(x)}{dx^\nu}$, where n is an integer value and ν is a real value.

Several methods have been extensively explored in the literature for the design and implementation of fractional order differentiator in discrete-time domain. Traditionally, discretization method is used to design FODs which includes mapping of fractional operator from s-domain to z-domain. It can be classified as direct and indirect discretization. In case of indirect discretization, transformation methods (bilinear transform, Al-Alaoui transform) are applied for s-to-z domain mapping of designed FODs [5]. Whereas, direct discretization methods includes the application of the series expansion of Euler operator, Tustin operator, numerical integration-based methods [6–8], continuous fractional expansion (CFE) [9], etc. In 2003, Chen and Moore have proposed discretized mathematical models of half differentiator using Al-Alaoui and Tustin operator [7]. Chen and Vinagre also proposed an infinite impulse response (IIR) fractional order digital differentiator based on the direct discretization method considering Simpson integration rule and trapezoidal integration rule [8]. Vinagre et al. presented continued fraction expansion and recursive Tustin transformation methods for discretizing the continuous-time FOD in [6]. Tseng presented the design of FOD by utilizing fractional sample delay which leads to the improved results at higher frequency region [10]. The digital rational function of continuous FODs are obtained using Pade, Prony and Shanks techniques by least-square estimation [11]. The discretized mathematical models of FOD using higher order operators like Schneider operator and

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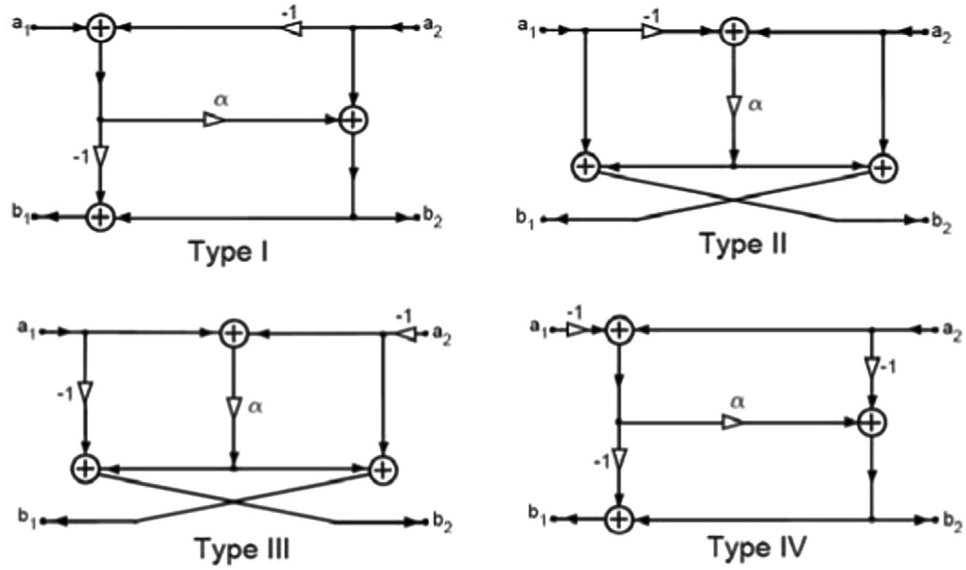

Fig. 1. Four equivalent structures of the two-port adaptor.

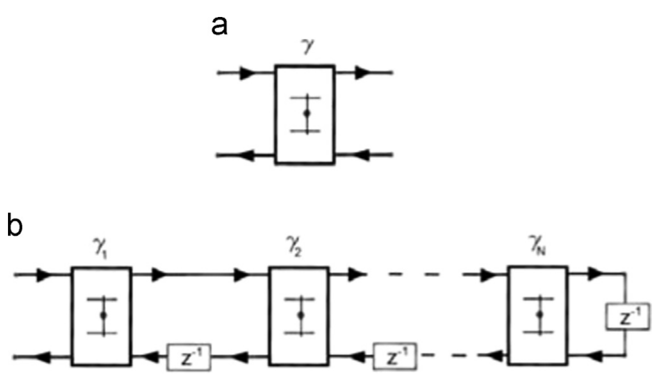

Fig. 2. (a) An adaptor symbol, (b) interconnection between adaptors.

Table 1
Type of adaptors.

Adaptor type	γ range	α/γ conversion
Type I	$\frac{1}{2} < \gamma < 1$	$\alpha = 1 - \gamma$
Type II	$0 < \gamma \le \frac{1}{2}$	$\alpha = \gamma$
Type III	$-\frac{1}{2} \le \gamma < 0$	$\alpha = -\gamma$
Type IV	$-1 < \gamma < -\frac{1}{2}$	$\alpha = 1 + \gamma$

Al-Alaoui-SKG rule is introduced in [12]. Romero et al. have applied Chebyshev polynomials to attain discrete-time approximations of FOD [13]. In the design of discrete FODs, applications of optimization techniques such as PSO, GA, etc. have emerged out as a recent trend. In 2014, improved FOD models are proposed based on optimization of digital differentiators using evolutionary algorithms such as PSO, GA and hybrid PSO-GA optimization [14]. Rana et al. have applied Nelder Mead simplex algorithm for the designing of FOD operators by optimizing the coefficients obtained by binomial expansion [15]. Recently, Kumar and Rawat proposed a finite impulse response (FIR)-FOD by approximating the frequency response of the FIR system to an ideal FOD using cuckoo search algorithm [16].

Each method has its unique feature, but there is always a scope for a new methodology to improve the design of FOD and reduce its hardware requirements which is dealt in this paper. Here, instead of direct form of FIR or IIR system, a new and improved class of recursive system is reported, but before discussing that some design constraints of IIR system is discoursed. For designing of IIR-FOD, a set of numerator and denominator coefficients of IIR system are optimized by minimizing the error fitness functions. It complicates the search as computational complexity increases due to increased number of variables to be optimized and the constraints that need to be incorporated to ensure stability of the IIR system. IIR systems also have limitations like sensitivity to word-length and coefficient round-off errors which make their implementation tricky. When the implementation of the system is considered, structural realization plays an important role and the number of multiplier coefficients determines the efficiency. The diversity of IIR system realizations (direct forms, lattice, state-space, etc.) has taken into consideration in past [17]. Some of these structures are more sensitive to the quantization of the coefficients, whereas some of them are very robust to roundoff errors. The direct form structure of an Nth order IIR system based FOD will require $2N+1$ multiplier coefficients and it is known that multipliers are the most power consuming element which leads to increase in cost, power dissipation and hardware utilization. Due to the large number of multiplications involved in direct form structure, the hardware implementation gives rise to excessive area, delays and power consumption.

All these limitations of IIR system encouraged us to explore other possibilities for competent design of FOD, leading to wave digital filters (WDFs). WDFs were first introduced by Fettweis [18] in 1971. WDFs are closely related to classical filter networks [18], which distinguishes them from other types of digital filters leading to a new class of digital filters whose passivity as well as stability constraints can be easily maintained even with finite word-length constraint [18,19]. A wide variety of WDFs are available and utilized in digital systems. The theory, principles and properties of wave digital filters are well explained in various literatures [17,19]. A specific class of WDFs is called the lattice wave digital filter. It is known that the LWDF structures exhibit many attractive properties such as low coefficient sensitivity and consequently the low accuracy requirements for the register word length, higher dynamic range, higher overflow level, lower round-off noise, assurance of stability and good nonlinear properties under finite-arithmetic conditions where the effects of rounding, truncation and overflow are present [19,20]. In the past, LWDF structures were used for realizing lowpass-highpass filter, bandpass-bandstop filter and Hilbert transformers [21–23]. Their resulting structures are found to have minimum hardware, highly modular