ARTICLE IN PRESS

ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

Using the expected detection delay to assess the performance of different multivariate statistical process monitoring methods for multiplicative and drift faults

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ARTICLE INFO

Article history:
Received 18 July 2016
Received in revised form
18 September 2016
Accepted 15 November 2016

Keywords:
T^2-statistic
Q-statistic
Multiplicative faults
Drift faults
Performance assessment
Expected detection delay

ABSTRACT

Using the expected detection delay (EDD) index to measure the performance of multivariate statistical process monitoring (MSPM) methods for constant additive faults have been recently developed. This paper, based on a statistical investigation of the T^2- and *Q*-test statistics, extends the EDD index to the multiplicative and drift fault cases. As well, it is used to assess the performance of common MSPM methods that adopt these two test statistics. Based on how to use the measurement space, these methods can be divided into two groups, those which consider the complete measurement space, for example, principal component analysis-based methods, and those which only consider some subspace that reflects changes in key performance indicators, such as partial least squares-based methods. Furthermore, a generic form for them to use T^2- and *Q*-test statistics are given. With the extended EDD index, the performance of these methods to detect drift and multiplicative faults is assessed using both numerical simulations and the Tennessee Eastman process.

1. Introduction

Process maintenance and management require detailed process operating information to determine not only whether the process is operating normally, but also to determine the potential causes for any observed problems. In modern industrial plants, multi-dimensional, correlated process data are ubiquitous. The challenging issue is how to determine if the data are informative enough to monitor the process and which methods can be used to achieve this objective. One approach to this problem is through process monitoring and fault detection (PM-FD) that seeks to examine the information provided by routine operating data to determine the existence of problems and their probable root causes [1–3]. Early work in this field was performed by Walter Shewhart in the early 1920s, who developed Shewhart control charts that allowed for easy tracking of the reliability of telephone transmission systems [2]. Afterwards, this approach spread to other physical processes, where a n ormal distribution can be typically assumed. Shewhart charts are easy to use and create, but are limited to univariate monitoring which does not take into consideration any dependencies between the monitored variables. More recently, the approach has been extended and improved by incorporating advanced statistical methods [3] to develop the T^2- and *Q*-statistics, by which control charts can be extended to multivariate cases [4–6].

At the same time as the development of control charts was occurring, work in chemometrics led to the development of new data analysis methods, for example, principal component analysis (PCA) and partial least squares (PLS) [7–9]. Finally, in the early 1990s, the PLS and PCA methods were combined with T^2- and *Q*-statistics leading to the development of a new field of chemical process monitoring [10–12]. These methods are primarily called multivariate statistical process monitoring (MSPM) or data-driven methods [13], and can be structured in the process control framework as shown in Fig. 1. It is shown that they take all the information about the process components (actuators, sensors, controllers, and key performance indicator) in a process control loop into consideration. Thus, they can address different types of process faults. The general procedure for these methods is to develop analytical models of normal and faulty operating conditions, onto which the current process data can be projected to give a measure of its current performance [14]. The key difference between the PCA- and PLS-based methods is how they use the available data space. As can be seen in Fig. 1, PCA-based methods

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http://dx.doi.org/10.1016/j.isatra.2016.11.007
0019-0578/

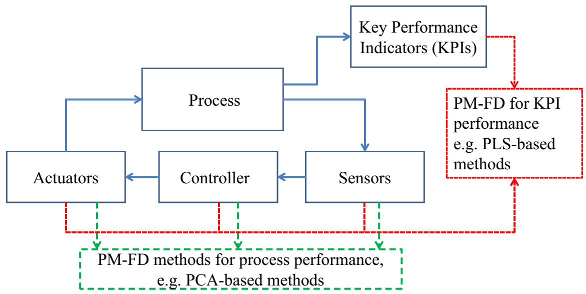

Fig. 1. Multivariate statistical process monitoring systems [13].

monitor the complete data space, while PLS-based methods monitor solely a subspace of the complete data space, commonly referred to as the key performance indicator (KPI)-related subspace [4]. It should be noted that the PCA model is indeed a subspace model that only considers the principal component subspace, while from the PM-FD viewpoint, PCA-based methods cover the complete data space. Due to the lack of first principles models, MSPM methods have been quickly adopted by chemical engineers [15–19]. As well, such methods have been applied to such areas as semiconductor, polymers, iron, and steel processes [20,21]. Although there are intensive uses of T^2 and Q statistics in MSPM literature, and some commercial uses were reported in practical applications, the study of developing a framework based on how the MSPM methods use the two statistics is rare [22–24]. In addition, even though these methods are reported to be practical in industrial application, few work has focused the attention on assessing their performance using statistical approaches.

In many industrial applications, MSPM methods are used to detect faults, of which the most common application is to additive faults, that is, those faults which change the mean value of the process. The implementation and assessment of these methods for multiplicative faults, which impact the variance or covariance parameters of a process variable has rarely been considered. In [25], Hao et al. showed, by comparing the original and current formulae for the T^2-statistic, that MSPM methods could be applied to multiplicative faults. However, greater details, specially from a statistical viewpoint, are required before such methods can be more readily accepted for detecting multiplicative faults. In addition to these methods, many other methods have been proposed for detecting multiplicative faults [26–28]. A third type of fault, the drift fault, which cause a slow change in the process parameters, has recently become a new area of focus. Such faults, if noticed early, can be mitigated before they adversely impact the system.

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In assessing the performance of MSPM methods, three different aspects are commonly considered: the false alarm rate (FAR), which examines the performance of the method in normal operating conditions; the fault detection rate (FDR), which considers the performance during faulty conditions; and the detection delay (DD), which measures the time delay before a fault is detected. These parameters can be defined either using a probabilistic approach [4,29,30] or using a numerical approximation approach [23]. In most cases, only the first two metrics are considered [15,21], while using all three is rarer [23,31]. Recently, the DD metric was extended to the case of stochastic systems to give an expected DD (EDD) index, which was then used, together with the FDR, to evaluate MPSM methods when detecting constant faults [22]. Given a constant fault and a method to be examined, EDD is capable of determining the expected detection delay time, but its application is limited to constant additive faults. The extension of this approach to multiplicative and drift faults has major practical interests.

Therefore, based on above motivations, the objectives of this paper are:

- to revisit the definitions of T^2- and Q-statistics, FDR, and EDD;
- to group the commonly used MSPM methods that involve the T^2- and Q-statistics;
- to extend the EDD index to drift and multiplicative faults;
- to evaluate the performance of the examined MSPM methods for detecting drift and multiplicative faults using EDD; and
- to demonstrate the results using numerical case studies and the Tennessee Eastman (TE) benchmark process.

The paper is organised as follows. Section 2 introduces the definitions of T^2- and Q-statistics, FDR as well as the newly developed performance evaluation index, EDD. In Section 3, a unified framework for MSPM methods that use the two statistics is presented. Sections 4 and 5 focus on the calculation form of EDD for all MSPM methods. In Section 4, the calculation of FDR is firstly addressed, while in Section 5, the calculation of EDD based on FDR is presented. The proposed methods are tested using a numerical case and the TE benchmark process. Finally, Section 7 presents the key conclusions and future direction.

Notation: Let $\mathbb{R}^m$ be the m-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ be the set of $n \times m$ real matrices; $(\cdot)^\dagger$ denote the pseudo-inverse; I_m be the $m \times m$-dimensional unit matrix; $tr(\cdot)$ represent the trace of a matrix; $\oplus$ denote the direct sum of two vector-spanned spaces; $P^\perp$ denote the orthogonal complement of P; $\vee$ and $\wedge$ represent logical'or' and 'and'; $x \sim \mathcal{N}_\Upsilon(\mu_x, \Sigma_x)$ represent a Υ-variate Gaussian-distributed vector x with mean vector μ_x and covariance matrix Σ_x; $\chi^2_{l,\alpha}$ be the χ^2-distribution with$l$degrees of freedom; $\chi^2_l(\lambda)$ be the noncentral χ^2 distribution with$l$degrees of freedom and noncentrality parameter λ; Let $\mathrm{prob}\left(\chi^2 > \chi^2_{l,\alpha}\right) = \alpha$ represent the probability that $\chi^2 > \chi^2_{l,\alpha}$ equals α.

2. Definitions of J_{T^2}, J_Q, FDR and EDD

Let $y_{obs} \in \mathbb{R}^m$ denote the process measurement vector. It is assumed that $y_{obs} \sim \mathcal{N}_m\left(\mu_y, \Sigma_y\right)$, μ and Σ_y are unknown beforehand. Given sufficient samples denoted as $Y_{obs} = \left[y_{obs,1}, \ldots, y_{obs,N}\right] \in \mathbb{R}^{m\times N}$, μ_y and Σ_y can be unbiasedly estimated using $\mu_y \approx (1/N)\sum_{i=1}^N y_{obs,i}$, $\Sigma_y \approx (1/(N-1))\sum_{i=1}^N \left(y_{obs,i} - \mu_y\right)\left(y_{obs,i} - \mu_y\right)^T$. For the sake of simplicity, the mean-centered vector $y \sim \mathcal{N}_m\left(0, \Sigma_y\right)$ is used. Designing FD methods for monitoring y consists of (1) defining the detection (test) statistic J and the corresponding threshold J_{th}; (2) comparing the online realization of J against J_{th} to make the decision: faulty or fault-free. The commonly used statistics are the T^2- and Q-statistics [6].

Given the covariance structure, Σ_y, and routine measurement vector, y, the two statistics are defined [4,6]