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Comput. Methods Appl. Mech. Engrg. 194 (2005) 2001–2017
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

An upwind local RBF-DQ method for simulation of inviscid compressible flows

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Received 12 February 2004; received in revised form 19 May 2004; accepted 5 July 2004

Abstract

In this paper, an upwind local radial basis function-based differential quadrature (RBF-DQ) scheme is presented for simulation of inviscid compressible flows with shock wave. RBF-DQ is a naturally mesh-free method. The scheme consists of two parts. The first part is to use the local RBF-DQ method to discretize the Euler equation in conservative, differential form on a set of scattered nodes. The second part is to apply the upwind method to evaluate the flux at the mid-point between the reference knot and its supporting knots. The proposed scheme is validated by its application to simulate the supersonic flow in a symmetric, convergent channel and the shock tube problem. The obtained numerical results agree very well with the theoretical data.

Keywords: Radial basis function; Meshless method; Upwind scheme; Inviscid compressible flow

1. Introduction

For the practical applications characterized by complex geometries and/or complex physics, one of major difficulties in computational fluid dynamics (CFD) is the generation of suitable meshes. It is very expensive to generate a mesh for a flow problem involving complex configuration, in the sense of human labor and CPU time. In order to reduce the high cost of mesh generation, a lot of numerical schemes have been proposed in the past two decades. Among them, one class of schemes attempts to use very simple

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0045-7825/$ - see front matter
doi:10.1016/j.cma.2004.07.008

technique to generate the grid covering the domain. The generated grid can have only one single type of mesh, or is composed of hybrid meshes. A typical numerical scheme of this class is the Cartesian mesh method [1–3], in which the Cartesian mesh is employed and the irregular geometries are treated as a special boundary embedded in the mesh. Since the computational cost used for the generation of Cartesian mesh is negligible as compared with that required by the body-fitted or unstructured mesh, the method is thought to be very efficient. Another class of schemes completely discards the mesh dependence during the process of spatial discretization. In other words, they are able to approximate the unknown function or its derivatives on a set of scattered nodes within the local support. Since they do not require the mesh to do spatial discretization, they are usually referred to as *mesh-free* or *meshless* methods. The *mesh-free* property of these schemes not only suggests that they do not rely on the mesh, but also implies that they can be applied on any kind of meshes or their hybrids.

In the past decade, the so-called *mesh-free* methods have become one of the hottest research areas in computational mechanics. In the viewpoint of kernel interpolation/approximation techniques, the mesh-free methods to date can be grouped into two categories. One is based on the least-square (LS) technique or its equivalents. This interpolation scheme is adopted by many popular mesh-free methods [4–10]. The least-squares technique allows an optimized approximation derived from an over-determined set of equations, and generally the resultant coefficient matrix has good properties such as positive, symmetric, and definite. Thus, the problem of singular coefficient matrix can be circumvented by means of using more local supporting points (more than the unknowns). Another is to use the radial basis functions (RBFs) as the interpolants. Employed as the base functions for multi-variate data interpolation, RBFs have been shown able to construct a scheme with favorable properties such as high efficiency and good quality. There are two notable benefits we can enjoy by using RBFs to solve partial differential equations. One is the naturally mesh-free property, which refers to their natural ability of dealing with scattered data. The other is that the RBF-based scheme generally has higher-order accuracy than the typical finite difference schemes on the scattered nodes. Motivated by these advantages, many researchers cast their sights on the development of RBFs-based methods and a lot of literatures are available now in this field [11–20]. Recently, a local RBF-based differential quadrature (RBF-DQ) method was proposed by Shu et al. [21]. Some examples of applying the local RBF-DQ method to simulate incompressible viscous flows were shown in [21]. In this paper, a simple and efficient solution strategy is designed for the local RBF-DQ method to simulate the compressible flow with shock wave governed by the Euler equations.

In the field of compressible flow simulation by mesh-free methods, some schemes have been proposed and studied in the past. Among them, least square kinetic upwind (LSKU) scheme was proposed by Deshpande et al. [22], in which one-sided local supporting points are employed to bring in the artificial dissipation. As compared with the work of Deshpande et al. [22], Lohner et al. [23] used the finite point method, and adopted a cloud of unbiased supporting points to do the spatial discretization. In their scheme, the artificial dissipation was introduced by modifying the flux function according to the local physics. By using the center-spaced local support, Sridar and Balakrishnan proposed LSFD-U method [24]. However, different from the scheme of Lohner et al. [23], the modification of flux function follows the idea of finite volume method, in which flux is modified along the direction from the central node to its neighbors. Since additional unknowns are involved, the LSFD-U scheme generally needs more local supporting points to evaluate the flux than finite point scheme. It should be noted that all the schemes mentioned above for the compressible flow computation adopt least-square technique to achieve the mesh-free property. It is of great interest to examine the performance of the local RBF-DQ method in this field. In general, the local RBF-DQ method requires less supporting points to achieve the same order of accuracy in the smooth flow region than the least-square-based schemes. This implies that the local RBF-DQ scheme may have better efficiency.

The algorithm of present mesh-free Euler solver is composed of two parts: one is the derivative approximation scheme, i.e., local RBF-DQ method; the other is the mesh-free upwind method for flux evaluation.