ISA Transactions 53 (2014) 1716–1730

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Delay-range-dependent chaos synchronization approach under varying time-lags and delayed nonlinear coupling

Muhammad Hamad Zaheer [a,1], Muhammad Rehan [a,*], Ghulam Mustafa [a,1], Muhammad Ashraf [b,2]

[a] Department of Electrical Engineering, Pakistan Institute of Engineering and Applied Sciences (PIEAS), P. O. Box 45650, Islamabad, Pakistan
[b] Department of Electronics Engineering, Mohammad Ali Jinnah University, Islamabad, Pakistan

ARTICLE INFO

Article history:
Received 27 March 2014
Received in revised form
28 August 2014
Accepted 17 September 2014
Available online 13 October 2014
This paper was recommended for publication by Mohammad Haeri

Keywords:
State feedback control
Chaos synchronization
Delay-range-dependency
Nonlinear time-delay coupling
Time-varying delays
Lyapunov–Krasovskii functional

ABSTRACT

This paper proposes a novel state feedback delay-range-dependent control approach for chaos synchronization in coupled nonlinear time-delay systems. The coupling between two systems is esteemed to be nonlinear subject to time-lags. Time-varying nature of both the intrinsic and the coupling delays is incorporated to broad scope of the present study for a better-quality synchronization controller synthesis. Lyapunov–Krasovskii (LK) functional is employed to derive delay-range-dependent conditions that can be solved by means of the conventional linear matrix inequality (LMI)-tools. The resultant control approach for chaos synchronization of the master–slave time-delay systems considers non-zero lower bound of the intrinsic as well as the coupling time-delays. Further, the delay-dependent synchronization condition has been established as a special case of the proposed LK functional treatment. Furthermore, a delay-range-dependent condition, independent of the delay-rate, has been provided to address the situation when upper bound of the delay-derivative is unknown. A robust state feedback control methodology is formulated for synchronization of the time-delay chaotic networks against the L_2 norm bounded perturbations by minimizing the L_2 gain from the disturbance to the synchronization error. Numerical simulation results are provided for the time-delay chaotic networks to show effectiveness of the proposed delay-range-dependent chaos synchronization methodologies.

1. Introduction

Chaos synchronization plays an essential role in physical, biomedical and engineering systems owing to its necessity in biological and natural processes as well as due to its vast applications in measuring brain activity, secure communications, chemical reactions, harmonic oscillator synthesis and image processing [1–11]. An important characteristic of the chaotic systems is their sensitivity to perturbations as well as initial conditions; accordingly, variations in time-delays should be carefully accounted for the synchronization controller synthesis studies. Time-delays are inherently present in the dynamics of chaotic systems and are also encountered due to the distant communication of coupled oscillators. These intrinsic and coupling time-lags can complicate the synchronization controller design by causing oscillations and instability in the closed-loop system response; therefore, much of the recent research is devoted to address chaos synchronization under time-delays [12–19].

Because of vast applications of the chaos synchronization in time-delay systems, several delay-independent and delay-dependent control strategies [14–26] under uncertainties, varying time-delays, input/output lags, and disturbances for both identical and different networks have been focused in the several past years. Recently, a delay-independent control methodology has been formulated for robust synchronization of the linearly coupled FitzHugh–Nagumo neurons with coupling time-delays [17]. Based on Kronecker product approach, synchronization conditions for coupled neural systems with coupling and leakage delays are derived [21]. Synchronization of two different Lur'e networks subject to time-varying delays and parametric uncertainties has been addressed in [24] via static state feedback and dynamic output feedback sliding mode control approaches. Delay-dependent synchronization of two different time-delay neural networks with monotonically non-decreasing and Lipschitz nonlinearities have been achieved in the work [25] by application of an integral sliding mode control strategy.

* Corresponding author. Tel.: +92 51 2207381~84x3443; fax: +92 51 2208070.
E-mail addresses: mhz.uet@gmail.com (M.H. Zaheer), rehanqau@gmail.com (M. Rehan), gm@pieas.edu.pk (G. Mustafa), ashraf@jinnah.edu.pk (M. Ashraf).
[1] Tel.: +92 51 2207381~84x3443; fax: +92 51 2208070.
[2] Tel.: +92 51 111878787; fax: +92 51 4486705.

http://dx.doi.org/10.1016/j.isatra.2014.09.007
0019-0578/

The delay-dependent control strategies are less conservative than the delay-independent ones, especially for systems with a small delay range; however, the main conservatism with the delay-dependent techniques is the consideration of the delay-range from zero to an upper bound. Additionally, the traditional delay-dependent approaches ignore some valuable terms while applying Lyapunov–Krasovskii (LK) functional, resulting conservative results [27]. Recently, more practical approaches for investigating stability of the linear time-delay systems under time-varying delays in a range, with lower bound not necessarily equal to zero, have been investigated through more general forms of the LK functional [27–29]. Because of complexity of the chaotic oscillators, these delay-range-dependent techniques have not been employed to a great extent for controlled chaos synchronization, apart from some exceptional recent works on Lur'e systems, chaotic time-delay structures, and delayed neural networks [30–34].

In this paper, synchronization of nonlinearly coupled chaotic systems with both the intrinsic and the coupling time-varying delays is explored. A Lyapunov–Krasovskii functional based less conservative synchronization condition, ensuring asymptotic convergence of the synchronization error to the origin, considering non-zero lower bounds of the time-delays, is derived by utilizing the state feedback delay-range-dependent control approach. To the best of our knowledge, the delay-range-dependent synchronization methodology for coupled chaotic oscillators with intrinsic and coupling time-delays has been addressed for the first time. Further, the nonlinear characteristic of coupling terms as well as the time-varying nature of delays is incorporated to investigate a matter-of-fact synchronization problem. Furthermore, the type of systems considered in the present approach differs from the traditionalistic synchronization schemes [30–34]. A delay-dependent approach is derived as a specific case of the present delay-range-dependent synchronization control action. A delay-rate-independent synchronization condition, owing to its importance in the field of chaos synchronization, is provided by application of the proposed LK functional treatment. In addition, the proposed control scheme is modified by determining an upper bound on the L_2 gain from the disturbance vector to the synchronization error to achieve robustness against perturbations. The resultant control laws for the delay-range-dependent chaos synchronization can be numerically determined by solving linear matrix inequalities (LMIs)-based algorithms based on the cone complementary linearization approaches. Simulation results of the proposed control methodologies are provided for synchronization of the coupled time-delay chaotic oscillators in the absence and presence of perturbations.

This paper is organized as follows: Section 2 provides system description. The proposed delay-range-dependent control strategy for chaos synchronization is addressed in Section 3 under different circumstances. Simulation results are detailed in Section 4. Section 5 draws conclusions of the study.

Standard notation is used throughout the paper. The matrix inequality $Q>0$ (or $Q \geq 0$) for a symmetric matrix Q implies that the matrix is positive-definite (or semi-positive-definite). Transpose of a matrix A is mentioned as A^T. The Euclidean norm for a vector z is shown by $||z||$. The L_2 norm of a vector z is expressed by $||z||_2$, where $||\cdot||_2 = \left(\int_0^\infty ||\cdot||^2 dt\right)^{1/2}$. The L_2 gain for a system with an input vector w and an output vector e is defined as $\sup_{\|w\|_2 \neq 0}\left(\|e\|_2/\|w\|_2\right)$. For matrices $X_1, X_2, \ldots, X_n$, the notation $\text{diag}(X_1, X_2, \ldots, X_n)$ represents a block-diagonal matrix with entry X_i at its ith diagonal block.

2. System description

Consider the following coupled master–slave systems:

$$\dot{x}_m(t) = Ax_m(t) + A_1 x_m(t-\tau_1(t)) + f(t, x_m(t)) + g(t, x_m(t-\tau_1(t))) - h(t, x_s(t-\tau_2(t))) + Dd_1(t), x_m(t) = \phi_1(t), \quad t \in \left[-\max(\tau_{12}, \tau_{22}) \quad 0\right], \tag{1}$$

$$\dot{x}_s(t) = Ax_s(t) + A_1 x_s(t-\tau_1(t)) + f(t, x_s(t)) + g(t, x_s(t-\tau_1(t))) - h(t, x_m(t-\tau_2(t))) + Dd_2(t) + Bu, x_s(t) = \phi_2(t), \quad t \in \left[-\max(\tau_{12}, \tau_{22}) \quad 0\right], \tag{2}$$

where $x_m \in R^n$ and $x_s \in R^n$ are the state vectors for the master and the slave systems, respectively. The control input for synchronization of the systems (1) and (2) is denoted by $u \in R^p$. The L_2 norm bounded time-varying disturbances acting on the master and the slave systems are represented by $d_1(t) \in R^q$ and $d_2(t) \in R^q$, respectively. The linear parts in dynamics of the master-slave systems are represented by constant matrices A, A_1, B, and D, having appropriate dimensions. The vector functions $f(t, x(t))$ and $g(t, x(t-\tau_1(t)))$ represent nonlinear dynamics of the chaotic oscillators. Delayed coupling effects for the master–slave chaotic oscillators are denoted by $h(t, x(t-\tau_2(t)))$. The terms $\tau_1(t)$ and $\tau_2(t)$ are representing the time-varying delays, arising due to the inherent modeling of the master–slave systems and appearing owing to the time-delay coupling between the chaotic oscillators, respectively, that satisfy

$$\begin{aligned} &0 \leq \tau_{i1} \leq \tau_i(t) \leq \tau_{i2}, \\ &\dot{\tau}_i(t) \leq \mu_i, \end{aligned} \tag{3}$$

for $i = 1, 2$. In general, the function $h(t, x(t))$ can be used to represent the strength of channel between two oscillators. The net effect of the master and the slave systems to the channel can be described by nonlinear terms $h(t, x_m(t)) - h(t, x_s(t-\tau_2(t)))$ and $h(t, x_s(t)) - h(t, x_m(t-\tau_2(t)))$, respectively. Note that $h(t, x_m(t))$ or $h(t, x_s(t))$ can be included into $f(t, x(t))$ without loss of generality (that is, $f(t, x(t)) = \bar{f}(t, x(t)) + h(t, x(t))$, where $\bar{f}(t, x(t))$ denotes the delay-free nonlinear dynamics of the chaotic oscillator by ignoring the channel strength) to reduce the mathematical complexity as in the present case. The time-varying vector-valued initial conditions are denoted by $\phi_1(t)$ and $\phi_2(t)$ for the master and the slave oscillators, respectively.

Assumption 1. The nonlinear functions $f(t, x(t))$, $g(t, x(t-\tau_1(t)))$ and $h(t, x(t-\tau_2(t)))$ satisfy the inequalities

$$||f(t, x_m(t)) - f(t, x_s(t))|| \leq ||L_1(x_m(t) - x_s(t))||, \tag{4}$$

$$||g(t, x_m(t-\tau_1)) - g(t, x_s(t-\tau_1))|| \leq ||L_2(x_m(t-\tau_1) - x_s(t-\tau_1))||, \tag{5}$$

$$||h(t, x_m(t-\tau_2)) - h(t, x_s(t-\tau_2))|| \leq ||L_3(x_m(t-\tau_2) - x_s(t-\tau_2))|| \tag{6}$$

where L_1, L_2 and L_3 are constant matrices of appropriate dimensions.