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Stabilization strategies of a general nonlinear car-following model with varying reaction-time delay of the drivers

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ABSTRACT

In this paper, the stabilization strategies of a general nonlinear car-following model with reaction-time delay of the drivers are investigated. The reaction-time delay of the driver is time varying and bounded. By using the Lyapunov stability theory, the sufficient condition for the existence of the state feedback control strategy for the stability of the car-following model is given in the form of linear matrix inequality, under which the traffic jam can be well suppressed with respect to the varying reaction-time delay. Moreover, by considering the external disturbance for the running cars, the robust state feedback control strategy is designed, which ensures robust stability and a smaller prescribed H_∞ disturbance attenuation level for the traffic flow. Numerical examples are given to illustrate the effectiveness of the proposed methods.

1. Introduction

Traffic flow problem is an important research topic and many traffic flow models are proposed to understand the complex dynamics of traffic flow [1–5]. In particular, Bando et al. [3] proposed a car-following traffic model and derived a simple stability condition, in which the vehicle equation is governed by an optimal velocity function which depends on the headway distance. This model is called optimal velocity (OV) traffic model that describes the stop-and-go traffic characteristic and the traffic jam phenomenon of the real traffic flow. Stimulated by the above model, many new car-following traffic models were proposed to understand the complex characteristics of traffic flow. In [6], an improved traffic model was proposed to cope with particular situations like vehicles approaching standing cars. A coupled map car-following model with decentralized delayed feedback control scheme was proposed in [7] to suppress the traffic jam, which describes the dynamical behavior of a group of road vehicles traveling on a single lane without overtaking. In [8], a full velocity difference car-following model was studied, which considered both negative and positive velocity difference, and given a better description of starting process. Zhao and Gao [9] presented another simple strategy to suppress the congested state in the traffic system, in which the control signal incorporates the effect of the velocity difference between the preceding and the considering vehicles. By considering the navigation in modern traffic, a two velocity difference model for a car following theory was developed in [10], and the property of the model was investigated by using linear and nonlinear analyses. An extended car-following model which takes into account the honk effect was proposed in [11], which shows that the honk effect improves the stability of traffic flow. In [12], a modified coupled map car-following model was proposed, in which two successive vehicle headways in front of the considering vehicle were incorporated into the optimal velocity function, and a new control scheme was presented to suppress the traffic jam. With the consideration of varying road condition, a new car-following model was developed in [13], which shows that the road condition has great influences on uniform flow. In [14], a new anticipation optimal velocity model was proposed for car following theory on single lane byconsidering anticipation effect, and the linear stability condition was derived by the linear stability analysis.

Time delays play a major role in traffic behavior due to the time needed by human operators in sensing velocity and position variations of the vehicles in the traffic [4,15]. The optimal velocity model by incorporating the time-delay effect was proposed in [16], in which the time delay was constant and the properties of congestion and the delay time of car motion were analyzed. The study in [17] offered two modified optimal velocity models with time delays in order to create reasonable traffic models that better match the reality. In [18], the bifurcations and multiple traffic jams in a car-following model with reaction time delay were considered, and the stability of the uniform-flow equilibrium was studied. In [19], the finite reaction times were considered for the time-continuous microscopic traffic models, which showed that

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the destabilizing effects of reaction times can be compensated by spatial and temporal anticipation. By considering the delay of the drivers response in sensing headway, an extended car-following model was proposed in [20] to describe the traffic, and the neutral stability line and the critical point were obtained by using the linear stability theory.

Although there were numerous works on the analysis of the car-following model with reaction time delays, the existing literatures mainly modeled the time-delayed actions of the drivers by a discrete delay. In practice, the reaction time-delay of the drivers should be time varying and bounded. It is more realistic to consider the time delay as time varying. The stability and stabilization problems of the time-varying delay systems were studied in [21–23]. However, little literature can be found to deal with the nonlinear optimal velocity (OV) traffic model with varying reaction-time delay. Motivated by the above discussions, in this paper, we consider the stabilization strategies of a general nonlinear car-following model with varying reaction-time delay of the drivers. By using the Lyapunov functions theory, the sufficient condition for the existence of the state feedback control strategy for the stability of the car-following model is given in the form of linear matrix inequalities, under which the traffic jam can be well suppressed with respect to the varying reaction-time delay. Additionally, the running vehicles will inevitably suffer from the uncertain external disturbances, such as irregular surfaces, bad weather, and equipment failure. To guarantee all the cars run orderly with the desired velocity with respect to the uncertain external disturbances, similar to the robust tracking control problem of nonlinear systems [24–29], the robust control method can be better applied to study the stability criteria for the car-following model with respect to the uncertain external disturbances. The H_∞ control method can not only guarantee the stability of the car-following model, but also guarantee a smaller prescribed disturbance attenuation level with respect to the external disturbances [30–33]. Then based on the H_∞ control method, we design the robust state feedback control strategy, which ensures robust stability and a smaller prescribed H_∞ disturbance attenuation level for the traffic flow. Numerical examples are given to illustrate the effectiveness of the proposed methods. From the numerical examples, we can observe that the proposed control methods can be effectively used to suppress traffic jam with respect to the varying reaction-time delay. Moreover, the proposed robust control methods effectively suppress the uncertain external disturbance to the stability of traffic flow.

The rest of this paper is organized as follows. In Section 2, a general nonlinear optimal car-following model with varying reaction-time delay is presented. In Section 3, the stabilization strategies of the nonlinear car-following model with varying reaction-time delay are considered. In Section 4, numerical examples are provided to demonstrate the effectiveness of our theoretical results. We conclude this paper in Section 5.

2. Optimal car-following model

Let us consider a general nonlinear optimal velocity traffic model with varying reaction-time delay of the drivers under an open boundary condition. The leading vehicle is supposed to be running with a constant velocity $v_0 > 0$, and the motion of the leading vehicle is described as follows:

$$x_0(t) = x_0(0) + v_0 t, \tag{1}$$

where $x_0(t) > 0$ is the position of the leading vehicle at time t, $x_0(0) > 0$ is its initial position.

Assume that the leading vehicle is not influenced by other following vehicles. By considering reaction-time delay of the drivers, the motions of the following vehicles group are described by a nonlinear dynamical delayed equation as follows:

$$\begin{cases} \dot{y}_i(t) = v_{i-1}(t) - v_i(t), \\ \dot{v}_i(t) = a_i(F(y_i(t-\tau(t))) - v_i(t)) + u_i(t), \quad i = 1, 2, \ldots, n, \end{cases} \tag{2}$$

where $y_i(t) > 0$ is the headway distance between the $i-1$-th and i-th vehicles, $v_i(t)$ is the velocity of the ith vehicle, $a_i > 0$ denotes the sensitivity of the driver, the different $a_i > 0$ means that each driver has a different sensitivity, n is the number of following vehicles, $F(y_i(t-\tau(t)))$ is the OV function related to the behavior of individual drivers, which depends on the distance between the $(i-1)-$th and the i-th vehicles, $u_i(t)$ is the control input signal to be designed, $\tau(t)$ is the varying reaction-time delay of the drivers and satisfies that

$$0 \le \tau(t) \le h, \quad \dot{\tau}(t) \le d < 1, \tag{3}$$

where the bounds h and d are known constant scalars. h is the maximum reaction time delay due to the rational behavior of the driver. In addition, it is shown empirically that the variations of time delay of humans are very slow [34]. So it is reasonable to assume that $\dot{\tau}(t) \le d < 1$. Additionally, for the dynamical delayed equation (2), the initial condition is given as $y_i(t) = \phi_i(t)$, $t \in [-h, 0]$, $i = 1, 2, \ldots, n$, where $\phi_i(t)$ is a continuous and differentiable function.

The physical basis of the model (2) is the fact that the reaction time of the driver performing his decision is varying with the time, in contrast to the case that the reaction time is discrete, which can better match the reality to describe the behavior of individual drivers for the traffic dynamics.

Without loss of generality, we choose a general nonlinear OV function as follows:

$$F(y(t)) = \frac{v_{\max}}{2}(\tanh(y(t) - y_c) + \tanh(y_c)), \tag{4}$$

where $v_{\max}$ is the maximum velocity, y_c is the safe distance. Moreover, it is clear that the nonlinear OV function (4) satisfies that

$$\| F(y_1(t)) - F(y_2(t)) \| \le \frac{v_{\max}}{2} \| y_1(t) - y_2(t) \|, \quad \forall y_1(t), \ y_2(t) \in R^m. \tag{5}$$

The form of condition (5) holds for the most proposed nonlinear OV functions in the existing literatures. It is very easy to prove that for the most proposed OV functions, there exists some constant $\alpha > 0$, such that the following condition holds:

$$\| F(y_1(t)) - F(y_2(t)) \| \le \alpha \| y_1(t) - y_2(t) \|, \quad \forall y_1(t), \ y_2(t) \in R^m. \tag{6}$$

Consider that the leading vehicle runs constantly with velocity v_0. The car-following model (2) has the following steady state:

$$[y_i^*(t), v_i^*(t)]^T = [F^{-1}(v_0), v_0]^T, \quad i = 1, 2, \ldots, n, \tag{7}$$

which implies that, in the steady state, all the vehicles run orderly with velocity v_0 and headway distance $F^{-1}(v_0)$.

The objective of this paper is to design the state feedback control input signal $u_i(t)$ to guarantee that the following vehicles run orderly with velocity v_0 and headway distance $F^{-1}(v_0)$ by rejecting the effect of the varying reaction time delays, so as to effectively suppress the traffic jam.

To obtain the main results, the following lemmas are needed.

Lemma 2.1 (*Jensen's inequality, Gu et al. [35]*). *For any constant matrix $R > 0$, scalar $b > 0$, and vector function $x : [0, b] \to R^n$, one has*

$$b \int_0^b x^T(s) R x(s) \, ds \ge \left(\int_0^b x(s) \, ds \right)^T R \left(\int_0^b x(s) \, ds \right).$$

Lemma 2.2 (*Schur Complement, Boyd et al. [36]*). *The matrix inequality*

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & -S_{22} \end{bmatrix} < 0,$$