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Design of a robust model predictive controller with reduced computational complexity

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ABSTRACT

The practicality of robust model predictive control of systems with model uncertainties depends on the time consumed for solving a defined optimization problem. This paper presents a method for the computational complexity reduction in a robust model predictive control. First a scaled state vector is defined such that the objective function contours in the defined optimization problem become vertical or horizontal ellipses or circles, and then the control input is determined at each sampling time as a state feedback that minimizes the infinite horizon objective function by solving some linear matrix inequalities. The simulation results show that the number of iterations to solve the problem at each sampling interval is reduced while the control performance does not alter noticeably.

1. Introduction

Model predictive control (MPC) is widely adopted to control industrial processes. Though almost all processes are inherently nonlinear, most MPC applications are based on linearized or uncertain linear dynamic models [1,2]. One of the main reasons for this choice relates to the high on-line computational complexity which resulted from the direct use of nonlinear and non-convex programming techniques [3].

By using a linear model and a quadratic objective function, the convex quadratic program which resulted from the MPC algorithm can easily be solved but for some processes, nonlinear effects are significant and therefore they should be considered in the control design stage. In this case, a convex problem that is solved efficiently via semi-definite programming could be tailed for MPC for nonlinear systems, through linear dynamic approximation of the nonlinear process together with defining a quadratic bound on the approximation error [4]. Nevertheless, for on-line implementation of MPC strategies, effective computational methods should be employed. Since there are effective and powerful algorithms for solving linear matrix inequality (LMI) problems, minimizing the upper bound on the worst-case objective function defined in MPC could be transformed to a convex optimization problem involving LMIs [1,5–8]. Solving an LMI-based optimization problem is generally performed iteratively. If an algorithm be able to reduce the number of required iterations, the computation time would be reduced in accordance.

There are methods to reduce the computational complexity of optimization problems defined in an MPC algorithm. Combining some methods for improving the speed of model predictive control and considering some variations on the basic infeasible start a primal barrier method can be used for on-line optimization in the linear systems with disturbances that are independent identically distributed with known distribution [9]. The technique based on a robust model predictive control design, the stability enforcing constraint and the warm-start procedure are among the methods that provide guarantees on feasibility and stability for real time constraints for systems with both polytopic uncertainty and bounding disturbances in a compact and convex set [10]. In a linear conjugate gradient method, by defining scaled variables based on a transformation matrix that has elements corresponding to the conjugate directions, the contours of the objective function become ellipses whose axes are aligned with the coordinate directions. The minimum of such objective function could be found by performing one-dimensional minimizations along the coordinate directions. As a result, the number of iterations to reach the solution is generally reduced [11].

In this paper a new approach to reduce the computational complexity in the robust model predictive control (RMPC) is introduced. In the proposed algorithm, the nonlinear system is approximated by a linear model around an equilibrium point and the difference between the linear and the nonlinear models

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assumed to be locally Lipschitz. In the controller design stage, this nonlinear term (the linear approximation error) is replaced by a quadratic function of states (and/or inputs) which is determined using sum of squares (SoS) in an off-line manner.

At the beginning time step, a state feedback that minimizes the upper bound of an infinite horizon objective function is computed by solving a set of defined linear matrix inequalities. Then the region of feasibility for the defined problem is determined that is an invariant ellipsoid. For the remaining time steps a scaled state space is defined and the scaling matrix is computed by solving some nonlinear equations so that the contours of this region become vertical or horizontal ellipses or circles. Finally the problem is re-solved for the scaled system and the control law is designed as described before.

The rest of this paper is organized as follows. Section 2 presents some preliminaries. Section 3 describes the mathematical formulation of RMPC problem using LMIs. In Section 4, an SoS algorithm is presented to calculate a dominant quadratic function for the terms consist of the nonlinear parts of the system. Section 5 explains the proposed method for decreasing the computational complexity in RMPC. In Section 6 a numerical example is given to describe the design procedure and illustrate the effectiveness of the method. Finally Section 7 draws some conclusions.

2. Preliminaries

Consider a nonlinear discrete time system represented by

$$x(k+1)=f(x(k),u(k)), \tag{1}$$

where k is the discrete time index, $x(k)\in R^n$ is the state vector, $u(k)\in R^m$ is the control input, and $f(0,0)=0$. It is assumed that the working point is transferred to the origin. Let A and B be $\partial f/\partial x$ and $\partial f/\partial u$ at the working point. Then dynamic system (1) can be reformulated as

$$x(k+1)=Ax(k)+Bu(k)+\tilde{f}(x(k),u(k)), \tag{2}$$

where

$$\tilde{f}(x(k),u(k))=f(x(k),u(k))-Ax(k)-Bu(k) \tag{3}$$

is locally Lipschitz. The state and control variables are required to satisfy the following constraints

$$x(k+i|k)\in\overline{\mathbb{X}},u(k+i|k)\in\overline{\mathbb{U}},i\geq 0 \tag{4}$$

where $\overline{\mathbb{X}}$ and $\overline{\mathbb{U}}$ are compact sets in R^n and R^m, respectively, both containing the origin as an interior point. In order to design a state feedback control law $u(k+i|k)=F(k)x(k+i|k)$, $i\geq 0$ for the system in (1), one can consider minimizing the following objective function over an infinite prediction horizon

$$\min_{u(k+i|k)} \quad J(k)=\sum_{i=0}^{\infty}x(k+i|k)^TQx(k+i|k)+u(k+i|k)^TRu(k+i|k) \tag{5}$$

subject to

$$x(k+i|k)\in\overline{\mathbb{X}} \text{ and } u(k+i|k)\in\overline{\mathbb{U}}, i\geq 0, \tag{6}$$

where Q and R are symmetric positive definite weighting matrices. Let us introduce a candidate Lyapunov function for system (1) at sampling time k in the form $V(x(k|k))=x(k|k)^TPx(k|k)$ with $P>0$ and $V(0)=0$. Suppose that $V(x)$ satisfies the following constraint

$$\begin{aligned}&V(x(k+i+1|k))-V(x(k+i|k))\\&\quad\leq -x(k+i|k)^TQx(k+i|k)\\&\quad-u(k+i|k)^TRu(k+i|k)\end{aligned} \tag{7}$$

By summing both sides of (7) from $i=0$ to ∞ and then applying (5), it follows that

$$x(\infty|k)^TPx(\infty|k)-x(k|k)^TPx(k|k)\leq -J(k). \tag{8}$$

For asymptotic stability of the resulting closed loop system, $x(\infty|k)$ must be zero which implies that

$$J(k)\leq x(k|k)^TPx(k|k)\leq\gamma, \tag{9}$$

where γ is a positive scalar and represents the upper bound of $J(k)$.

Lemma 1. *(Schur complements): Let $H(x)=H(x)^T$, $O(x)=O(x)^T$, and $D(x)$ be affine in x. Then the LMI*

$$\begin{bmatrix} H(x) & D(x)\\ D(x)^T & O(x)\end{bmatrix}>0 \tag{10}$$

is equivalent to

$$O(x)>0,\quad H(x)-D(x)O(x)^{-1}D(x)^T>0 \tag{11-1}$$

or

$$H(x)>0,\quad O(x)-D(x)^TH(x)^{-1}D(x)>0. \tag{11-2}$$

3. Model predictive control using LMI

In this section, the problem formulation for the proposed robust MPC and its related LMI-based optimization are discussed. In this method, a state feedback control law $u(k+i|k)=F(k)x(k+i|k)$, $i\geq 0$ is designed so that the upper bound of $J(k)$ (γ) is minimized instead of $J(k)$ itself. The new problem is defined as follows:

$$\min_{u(k+i|k)} \max J(k)=\sum_{i=0}^{\infty}x(k+i|k)^TQx(k+i|k)+u(k+i|k)^TRu(k+i|k) \tag{12}$$

subject to

$$x(k+i|k)\in\overline{\mathbb{X}} \text{ and } u(k+i|k)\in\overline{\mathbb{U}}, i\geq 0.$$

Solution to this problem which will be obtained by solving a set of LMIs is the output of the proposed MPC law.

Theorem 1. *Let $x(k)=x(k|k)$ be the state of the system in (1) measured at sampling time k. Consider the Euclidean norm constraint on the control input in the form of $||u(k+i|k)||_2\leq u_{max}$, $i\geq 0$. Then, the state feedback matrix F in the control law that minimizes the upper bound $V(x(k|k))$ on the objective function at instant k is given by $F=YX^{-1}$ where $X>0$ and Y are obtained from the solution of the following optimization problem.*

$$\min_{\gamma,\beta,X,Y}\gamma \tag{13}$$

subject to

$$\begin{bmatrix} I & *\\ x(k) & X\end{bmatrix}\geq 0, \tag{14-1}$$

$$\begin{bmatrix} X & * & * & * & *\\ AX+BY & X & * & * & *\\ WX & 0 & \beta I & * & *\\ Q^{\frac{1}{2}}X & 0 & 0 & \gamma I & *\\ R^{\frac{1}{2}}Y & 0 & 0 & 0 & \gamma I\end{bmatrix}\geq 0, \tag{14-2}$$

$$X-\beta I\geq 0, \tag{14-3}$$

and

$$\begin{bmatrix} u_{max}^2 I & *\\ Y^T & X\end{bmatrix}\geq 0, \tag{14-4}$$