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Fault diagnosis viewed as a left invertibility problem

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ABSTRACT

This work deals with the fault diagnosis problem, some new properties are found using the *left invertibility condition* through the concept of differential output rank. Two schemes of nonlinear observers are used to estimate the fault signals for comparison purposes, one of these is a proportional reduced order observer and the other is a sliding mode observer. The methodology is tested in a real time implementation of a three-tank system.

1. Introduction

A fault can be considered as a process degradation or degradation of the equipment performance caused by the change in the physical characteristic of the process, the input process or the external conditions. Industrial control systems have to deal with faults, therefore, fault diagnosis is a very important subject in control theory. System diagnosis helps us to detect and estimate faults in a process. In other words, the task of diagnosis is, from measurements of outputs and inputs, to reconstruct the fault vector.

The fault detection and isolation problem have been studied for more than three decades, many papers dealing with this problem can be found, see for instance the surveys [1–4] and the books [5–7]. For the case of nonlinear systems a variety of approaches have been proposed [8–13], such as those based upon differential geometric methods [14,15], and on the other hand, alternative approaches based on an algebraic and differential framework can be found in [16–20].

Currently, the diagnosis problem is playing an important role in modern industrial processes. This has led control theory into a wide variety of model-based approaches which rely on descriptions via differential and/or difference equations, contrary to other standpoints developed mainly among computer scientist (see [18,19] and references therein). The primary objectives of fault diagnosis are fault detectability and isolability, i.e., the possible location and determination of the faults present in a system and the time of their occurrences. The tasks of fault detection and isolation are to be accomplished by measuring only the input and the output variables.

This paper focuses on the diagnosis of nonlinear systems and the goal is to determine malfunctions in the dynamics. In this communication, the outputs are mainly signals obtained from the sensors. Their number is important to know whether a system is diagnosable or not.

In this paper, the diagnosis problem is tackled as a left invertibility problem throughout the concept of differential output rank ρ. Two schemes of observers are proposed in order to estimate the fault signals, one of them is a reduced-order observer based on a free-model approach and another is a sliding-mode observer based on a Generalized Observability Canonical Form (GOCF) [18]. Both schemes are proved to possess asymptotic convergence properties.

The class of systems for which this methodology can be applied contains systems that depend on the inputs and their time derivatives in a polynomial form. The type of faults considered in this work is additive and bounded, however, the algebraic approach can also be used to deal with multiplicative faults.

These proposals are applied in this paper to a three-tank system [21]. The Amira DTS200 three-tank system [22] has been widely considered for the experimental fault diagnosis study, see for instance [15,17,23], even recently, one work based on the geometric approach has been reported [15]. We can also mention one previous work with the three-tank system using the differential algebraic

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approach [17]; in that work the authors only report a numerical simulation study and not a real-time experiment, also they only solve the simplest case in which three measured outputs are available to estimate two faults, that is to say, they do not analyze the minimal number of measurements to attack the diagnosis problem as we do in the present work. The intention of choosing the three-tank system example is to clarify the proposed methodology and to highlight the simplicity and flexibility of the present approach. The three-tank system is known as a system with parameter uncertainties, so this work also deals with these uncertainties by means of algebraic parameter estimation, considering that there is no simultaneous presence of uncertainty and faults, in the same way as it is considered in [17].

This paper is organized as follows. In Section 2, some definitions of differential algebra are given. In Section 3, we discuss the left invertibility condition and we present some examples. In Sections 4 and 5 we give a brief description of the proposed observers. In Section 6 the three-tank system is analyzed. Finally, in Section 7 we illustrate this methodology with some experimental results.

2. Some definitions

Some basic definitions are introduced. Further details can be found in [17,18] and references therein.

Definition 1. Let $\mathcal{L}$ and $\mathcal{K}$ be differential fields. A differential field extension $\mathcal{L}/\mathcal{K}$ is given by $\mathcal{K}$ and $\mathcal{L}$ such that: (1) $\mathcal{K}$ is a subfield of $\mathcal{L}$ and (2) the derivation of $\mathcal{K}$ is the restriction to $\mathcal{K}$ of the derivation of $\mathcal{L}$.

Example 1. $\mathbb{R}\langle e^t\rangle/\mathbb{R}$ is a differential field extension, where $\mathbb{R}\subseteq\mathbb{R}\langle e^t\rangle$. e^t being a solution of $\dot{x}-x=0$.

Definition 2. Let $\xi=(\xi_1,\xi_2,\dots,\xi_n)$ be a set of elements of $\mathcal{L}$. If it satisfies an algebraic differential equation $P(\xi,\dot{\xi},\ddot{\xi}\dots)=0$ with coefficients in $\mathcal{K}$ it is called differentially $\mathcal{K}-$algebraically dependent, otherwise ξ is called differentially $\mathcal{K}-$algebraically independent.

Definition 3. Any set of elements of $\mathcal{L}$ which is differentially $\mathcal{K}-$algebraically independent and maximal with respect to inclusion forms is a differential transcendence basis of $\mathcal{L}/\mathcal{K}$. Two such bases have the same cardinality. This is called the *differential transcendence degree* of $\mathcal{L}/\mathcal{K}$ and denoted by *diff tr d*$^\circ\,\mathcal{L}/\mathcal{K}$.

Definition 4. Let $\mathcal{G}$, $\mathcal{K}\langle u\rangle$ be differential fields. A nominal dynamic consists of a finitely generated differential algebraic extension $\mathcal{G}/\mathcal{K}\langle u\rangle$, $(\mathcal{G}=\mathcal{K}\langle u,\xi\rangle,\xi\in\mathcal{G})$. Any element of $\mathcal{G}$ satisfies an algebraic differential equation with coefficients over $\mathcal{K}$ in the components of u and their time derivatives.

Example 2. Consider the following differential equation:

$$\dot{u}^2y+4\ddot{u}=0$$

In this case, y is algebraic over $\mathcal{K}\langle u\rangle$, therefore, it can be seen as a dynamic of the form $\mathcal{K}\langle u,y\rangle/\mathcal{K}\langle u\rangle$ where $\mathcal{K}=\mathbb{R}$ and $y\in\mathcal{K}\langle u,y\rangle$.

Definition 5. Any unknown variable x in a dynamic is said to be *algebraically observable* with respect to $\mathcal{K}\langle u,y\rangle$ if x satisfies a differential algebraic equation with coefficients over $\mathcal{K}$ in the components of u, y and a finite number of their derivatives. Any dynamic with output y is said to be algebraically observable if, and only if, any state variable has this property.

Example 3. Let us consider the following system:

$$\begin{cases}\dot{x}_1=3x_1x_2^2+u_1\\ \dot{x}_2=x_1+x_2^3u_2\\ y=x_2,\end{cases}\tag{1}$$

since x_1 and x_2 satisfy the polynomials $x_1+y^3u_2-\dot{y}=0$ and $x_2-y=0$, respectively, then x_1,x_2 are algebraically observable over $\mathbb{R}\langle u,y\rangle$ and by applying Definition 5, system (1) is algebraically observable.

Definition 6. A fault is not a permitted deviation of at least one characteristic property or parameter of any process in relation to the development of the same parameter under normal conditions. Faults are defined as transcendent elements over $\mathcal{K}\langle u\rangle$, therefore, a system with the presence of faults is a differential transcendental extension, denoted as $\mathcal{K}\langle u,f,y\rangle/\mathcal{K}\langle u\rangle$, where f is a vector that includes the faults and their time derivatives.

Definition 7. Let $\mathcal{G}$, $\mathcal{K}\langle u\rangle$ be differential fields. A fault dynamic consists of a finitely generated differential algebraic extension $\mathcal{G}/\mathcal{K}\langle u,f\rangle$, $\mathcal{G}=\mathcal{K}\langle u,f,\xi\rangle$, $\xi\in\mathcal{G}$. Any element of $\mathcal{G}$ satisfies an algebraic differential equation with coefficients over $\mathcal{K}$ in the components of u,f and their time derivatives.

Definition 8 (*Algebraic observability condition*). A fault $f\in\mathcal{G}$ is said to be diagnosable if it is algebraically observable over $R\langle u,y\rangle$, i.e., if it is possible to estimate the fault from the available measurements of the system.

Let us consider the class of nonlinear systems with faults described by the following equation:

$$\begin{cases}\dot{x}(t)=A(x,\overline{u})\\ y(t)=h(x,\overline{u}),\end{cases}\tag{2}$$

where $x=(x_1,\dots,x_n)^T\in\mathbb{R}^n$ is a state vector, $u=(u_1,\dots,u_m)\in\mathbb{R}^m$ is a known input vector, $f=(f_1,\dots,f_\mu)\in\mathbb{R}^\mu$ is an unknown input vector, $\overline{u}=(u,f)\in\mathbb{R}^{m+\mu}$, $y(t)\in\mathbb{R}^p$ is the output vector. A and h are assumed to be analytical vector functions.

Example 4. Let us consider the nonlinear system with one fault (f_1) on the actuator and one fault (f_2) on the sensor of output y_1:

$$\begin{cases}\dot{x}_1=x_1x_2+f_1+u\\ \dot{x}_2=x_1\\ y_1=x_1+f_2\\ y_2=x_2.\end{cases}\tag{3}$$

Since f_1,f_2 satisfy the differential algebraic equations

$$\begin{aligned}&f_1-\ddot{y}_2+y_2\dot{y}_2+u=0\\ &f_2-y_1+\dot{y}_2=0\end{aligned}\tag{4}$$

the system (3) is diagnosable and the faults can be reconstructed from the knowledge of u, y and their time derivatives.

Remark 1. The diagnosability condition is independent of the observability of a system.

Example 5. Let us consider the system

$$\begin{cases}\dot{x}_1=x_1x_2+f+u\\ \dot{x}_2=x_1\\ \dot{x}_3=x_3f+u\\ y=x_2.\end{cases}\tag{5}$$

In this case f is diagnosable. However, x_3 is not algebraically observable.