ARTICLE IN PRESS

ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Modified active disturbance rejection control for time-delay systems

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ARTICLE INFO

Article history:
Received 30 May 2013
Received in revised form
26 August 2013
Accepted 12 September 2013

Keywords:
Active disturbance rejection
Time delay
Synchronization
Observer
Linear matrix inequality

ABSTRACT

Industrial processes are typically nonlinear, time-varying and uncertain, to which active disturbance rejection control (ADRC) has been shown to be an effective solution. The control design becomes even more challenging in the presence of time delay. In this paper, a novel modification of ADRC is proposed so that good disturbance rejection is achieved while maintaining system stability. The proposed design is shown to be more effective than the standard ADRC design for time-delay systems and is also a unified solution for stable, critical stable and unstable systems with time delay. Simulation and test results show the effectiveness and practicality of the proposed design. Linear matrix inequality (LMI) based stability analysis is provided as well.

1. Introduction

Most industrial processes, e.g. combustion, distillation, waste water treatment, are often treated as either first order plus time delay (FOPTD) systems or second order plus time delay (SOPTD) systems to simplify the controller design. The time delay, also known as dead time, is generally associated with the transportation of the material or energy in the processes [1]. In addition, it may be the result of an approximation of a higher order dynamics with a lower order one, which is not the main focus of this paper.

The control design for time-delay systems is very challenging due to the fact that the time delay introduces additional phase lag, which increases with frequency, to the system, which reduces the stability margin or simply destabilizes it. Hence the achievable closed-loop bandwidth is normally limited to $1/\tau$ [2], where τ is the time delay. The well-known Smith predictor [3] has been the main method of choice to deal with such systems, since it can increase the closed-loop bandwidth by removing the time delay from the loop. The prerequisite is an accurate system model is available; otherwise the high bandwidth may cause instability due to model uncertainties. Although the tracking performance is enhanced, the disturbance rejection performance of the original Smith predictor is quite limited. In addition, it cannot deal with time-delay systems which have right half plane poles. To improve its performance numerous efforts have been made to modify the original Smith predictor [4]. In particular, the control of integral processes with time delay seems to attract much attention [5–8]. Zhong et al. wrote a series of four papers on this topic [9–12], proposing a disturbance observer based approach. There are also efforts on finding a unified solution for stable, integral or even unstable time-delay systems [13,14]. All of the above Smith predictor based methods, however, start with a fairly good mathematical model of the system.

In the absence of such a model, the active disturbance rejection control (ADRC) [15], which is known for its ability to accommodate both external disturbances and internal uncertainties, seems to be a viable alternative. The essence of ADRC is to treat the whole effect of both external disturbances and internal uncertainties as total disturbance, then estimate it using the extended state observer (ESO) and cancel it out in the control law. The ADRC has been successfully applied to various applications [16–20] and recent theoretical analysis of it can be found in [21–24], but mainly for systems without time delay.

The application of ADRC to time-delay systems has been studied by other researchers [25,26] as well. Several methods was proposed in [25] to deal with time delay in the ADRC design. The first one is to ignore the time delay and design the ADRC for the dynamics without time delay. This leads to limited performance. The second method approximates the time delay with a first order dynamic using the relation $e^{-\tau s} \approx 1/(\tau s+1)$ and adopts a higher order ADRC design. Other methods try to predict either the system output or the control signal based on the Taylor series, i.e., $\theta(t+\tau) \approx \theta(t)+\dot{\theta}(t)\tau$, when the time delay τ is small. The ADRC design for a multivariable time-delay system is studied in [26], where the approximation method is adopted. The original nonlinear ADRC designs in [25,26], though provide a relatively satisfactory performance, are rather too complex for practical applications. It is the aim of this paper to

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http://dx.doi.org/10.1016/j.isatra.2013.09.013

provide an effective and relatively easy to implement ADRC solution to the prevailing industrial process control applications.

The paper is organized as follows. In Section 2, the regular ADRC design is first introduced followed by the proposed modification to it. In Section 3, simulation and experimental results are provided to demonstrate the effectiveness of the proposed design. Section 4 provides the stability analysis for the closed-loop system applying the proposed design and Section 5 concludes the paper.

2. Active disturbance rejection control

2.1. The regular design

A simple motion control problem is used to illustrate the regular ADRC design. Consider the following system dynamics.

$$m\ddot{y} = F + g(y, \dot{y}, t) + w(t) \quad (1)$$

where m is the mass, y is the position, F is the control force, w is the disturbance force, t is the time and $g(y, \dot{y}, t)$ is a nonlinear time-varying function of the position and velocity, which may correspond to nonlinear spring and friction forces. In the context of active disturbance rejection, the original system (1) is reformulated as

$$\ddot{y} = bu + f(y, \dot{y}, w, t) \quad (2)$$

where $b = 1/m$, $u = F$ and $f(y, \dot{y}, w, t) = (g(y, \dot{y}, t) + w(t))/m$ is called the total disturbance [15] which includes not only the external disturbances but also the unknown internal dynamics. Then the state vector of the system is defined as $\mathbf{x} = [x_1 \quad x_2 \quad x_3]^T = [y \quad \dot{y} \quad f]^T$, which has three components. Note that for a second order system the state vector is normally defined as $\mathbf{x} = [y \quad \dot{y}]^T$ with two components. Here $x_3 = f$, which is called the extended state representing the total disturbance, is augmented to the regular design.

The state space representation of Eq. (2) is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + b\mathbf{B}u + \mathbf{E}\dot{f}$$
$$y = \mathbf{C}\mathbf{x} \quad (3)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \text{ and } \mathbf{C} = [1 \quad 0 \quad 0].$$

An ESO is designed for system (3) accordingly as

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \hat{b}\mathbf{B}u + \mathbf{L}(x_1 - \hat{x}_1) \quad (4)$$

where $\hat{\mathbf{x}} = [\hat{x}_1 \quad \hat{x}_2 \quad \hat{x}_3]^T$ is the observer state vector which provides an estimation of the system state vector $\mathbf{x}$, $\hat{b}$ is an estimation of b, and $\mathbf{L} = [l_1 \quad l_2 \quad l_3]^T$ is the observer gain vector. The controller is designed as

$$u = \mathbf{K}(\mathbf{r} - \hat{\mathbf{x}})/\hat{b} \quad (5)$$

where $\mathbf{r} = [r \quad \dot{r} \quad \ddot{r}]^T$, r is the reference signal, and $\mathbf{K} = [k_1 \quad k_2 \quad 1]$ is the controller gain vector. In practice, $\dot{r}$ and $\ddot{r}$ are set to zero if they are either not available or unbounded.

According to the parameterization technique proposed in [27], the individual observer gains $l_i (i = 1, 2, 3)$ are selected such that all eigenvalues of $\mathbf{A} - \mathbf{LC}$ are placed at $-\omega_o$, and they are found to be $l_1 = 3\omega_o$, $l_2 = 3\omega_o^2$ and $l_3 = \omega_o^3$ in this case. Similarly, the individual controller gains $k_i (i = 1, 2)$ are selected such that all eigenvalues of matrix $\tilde{\mathbf{A}}_{2\times2}$ are placed at $-\omega_c$, where $\tilde{\mathbf{A}}_{2\times2}$ is defined as

$$\begin{bmatrix} \tilde{\mathbf{A}}_{2\times2} & 0 \\ 0 & 0 \end{bmatrix} = \mathbf{A} - \mathbf{BK}$$

and they are found to be $k_1 = \omega_c^2$ and $k_2 = 2\omega_c$. Above, ω_o and ω_c are referred to as observer and controller bandwidth respectively, and are the tuning parameters of the ADRC design.

2.2. Modified ADRC design to accommodate time delay

The modification to the regular ADRC design is straightforward and intuitive. A time delay block is added, as shown in Fig. 1, to delay the control signal before it goes into the extended state observer. Since the system output is already delayed due to the system dynamic, this will synchronize the signals that go into the observer and allow it to provide meaningful estimations of the delayed system states and delayed disturbances.

Remark: This synchronization only removes the time delay from the observer loop, unlike in the Smith predictor where the time delay is removed from the main loop. Hence the closed-loop bandwidth of the modified design is still limited. To improve the tracking performance, however, the feedforward control can be used as an alternative, as it has been shown to be effective in [28].

Compare to the regular ADRC design described in Section 2.1, the proposed ADRC can be implemented by replacing Eq. (4) with the following.

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \hat{b}\mathbf{B}u(t-\tau) + \mathbf{L}(x_1(t) - \hat{x}_1(t)) \quad (6)$$

Though the modification is simple, it enhances the regular ADRC design by increasing achievable observer bandwidth, which is the key for an accurate estimation of the total disturbance. With appropriate tuning, the proposed method also provides a unified solution to a variety of time-delay systems (with stable, critical stable, or unstable poles), as will be demonstrated in Section 3.

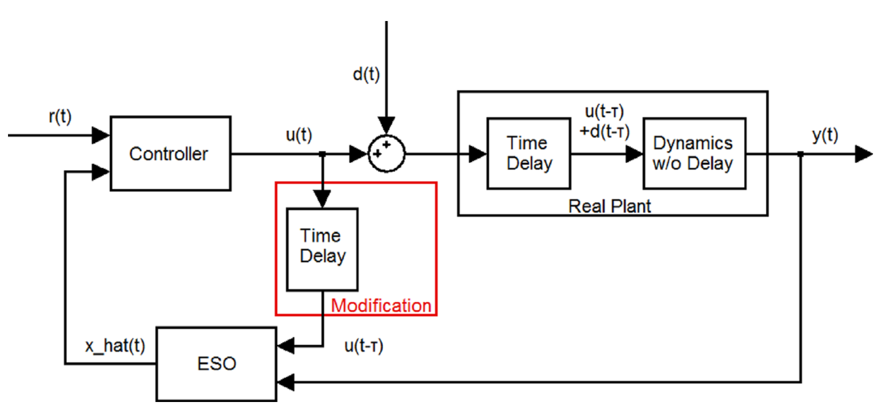

Fig. 1. Modified ADRC for time-delay systems.

Please cite this article as: Zhao S, Gao Z. Modified active disturbance rejection control for time-delay systems. ISA Transactions (2013), http://dx.doi.org/10.1016/j.isatra.2013.09.013