Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Linear active disturbance rejection control of underactuated systems: The case of the Furuta pendulum☆

M. Ramírez-Neria [a,1], H. Sira-Ramírez [a,1], R. Garrido-Moctezuma [a,1], A. Luviano-Juárez [b,*]

[a] Centro de Investigación y de Estudios Avanzados del I.P.N. (Cinvestav-IPN), Av. Instituto Politécnico Nacional 2508, Col. San Pedro Zacatenco, C.P. 07360 México, D.F. Apartado Postal, 14-740, 07000 México, D.F., Mexico
[b] Unidad Profesional Interdisciplinaria en Ingeniería y Tecnologías Avanzadas IPN, Av. Instituto Politécnico Nacional 2580 Col. Barrio La Laguna Ticomán C.P. 07340 México, D.F., Mexico

ARTICLE INFO

Article history:
Received 20 May 2013
Received in revised form
11 September 2013
Accepted 30 September 2013
This paper was recommended for publication by Dr. Jeff Pieper

Keywords:
Furuta pendulum
Differentially flat systems
Active disturbance rejection control
GPI observers

ABSTRACT

An Active Disturbance Rejection Control (ADRC) scheme is proposed for a trajectory tracking problem defined on a nonfeedback linearizable Furuta Pendulum example. A desired rest to rest angular position reference trajectory is to be tracked by the horizontal arm while the unactuated vertical pendulum arm stays around its unstable vertical position without falling down during the entire maneuver and long after it concludes. A linear observer-based linear controller of the ADRC type is designed on the basis of the flat tangent linearization of the system around an arbitrary equilibrium. The advantageous combination of flatness and the ADRC method makes it possible to on-line estimate and cancels the undesirable effects of the higher order nonlinearities disregarded by the linearization. These effects are triggered by fast horizontal arm tracking maneuvers driving the pendulum substantially away from the initial equilibrium point. Convincing experimental results, including a comparative test with a sliding mode controller, are presented.

1. Introduction

The control of underactuated systems represents a difficult and challenging problem, specially when experimental implementations of synthesized control solutions are required. This is due, aside of the effect of unmodeled dynamics and external forces, to the associated restrictions on the behavior of the non directly actuated variables [27] and the natural obstacle to linearizability exhibited by a large subclass of these systems. Underactuated systems are becoming popular in many sophisticated control applications, such as spacecraft, aerial robotic systems, underwater vehicles, locomotive systems, flexible robotics. Some possible advantages associated to such systems are cost reduction, lighter structures, smaller dimensions, among others (see [24] for a comprehensive treatment of this class of systems).

The Furuta pendulum [12], also called the rotational pendulum, is one of the most popular underactuated systems in academic laboratories around the world. The system is provided with one control input and it has two mechanical degrees of freedom. It consists of an actuated arm, which rotates in the horizontal plane; the actuated arm is joined to a non actuated pendulum which rotates loosely in a vertical plane perpendicular at the tip of the horizontal rotating arm. The system is quite nonlinear due to the gravitational forces, the Coriolis and centripetal forces [4] and the acceleration couplings. In addition, it is nonfeedback linearizable and it exhibits a lack of controllability in certain configurations [7]. The system represents a suitable platform for testing diverse linear and nonlinear control laws.

Traditional control problems associated with the Furuta pendulum are mainly of two kinds: (1) the problem of balancing up the vertical pendulum to the upper, unstable, position (swinging up) and (2) the stabilization around this position. Several methodologies have been proposed to solve the problem of swinging up and balancing the Furuta pendulum, these include the energy based swinging up control [2], passivity-based control [26], adaptive attractive ellipsoid methods [25], friction compensation controllers [34], extended state observer-based controllers [3], among others. In the study reported in [1], some different controllers for the Furuta pendulum were tested and compared to point out the principal advantages and drawbacks of the diverse control schemes. The study also considered the main physical limitations associated with the control of the pendulum, where among others, the possibility of control input saturations was specifically treated. Most of the

---

☆This work was supported by the Centro de Investigación y de Estudios Avanzados del I.P.N. (Cinvestav-IPN), México, D.F., México.

[*] Corresponding author. Tel.: +52 57296000.

E-mail addresses: mramirezn@ctrl.cinvestav.mx (M. Ramírez-Neria), hsira@cinvestav.mx (H. Sira-Ramírez), rgarrido@ctrl.cinvestav.mx (R. Garrido-Moctezuma), aluvianoj@ipn.mx, alberto.luviano@gmail.com (A. Luviano-Juárez).

[1] Tel.: +52 5747 3800.

0019-0578/$ - see front matter © 2013 ISA. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.isatra.2013.09.023

Please cite this article as: Ramírez-Neria M, et al. Linear active disturbance rejection control of underactuated systems: The case of the Furuta pendulum. ISA Transactions (2013), http://dx.doi.org/10.1016/j.isatra.2013.09.023

stabilizing schemes rely on the tangent linearization around the unstable equilibrium point, and demand robust linear stabilizing schemes [25].

The linearized model of the Furuta pendulum is differentially flat (i.e., it is controllable) with a physically measurable flat output. Thus, the problem of stabilization and tracking can be tackled from a combined perspective of flatness and Active Disturbance Rejection Control. Here, we propose the use of a linear decoupled extended observer, motivated by the structure revealed by flatness, in an active disturbance canceling scheme of the ADRC class. In general, ADRC dates back to the French engineer Poncelet (see [14]). Seminal work about a closely related technique, called Disturbance Accommodation Control, is due to Johnson [16]. Other variants of the ADRC idea are found as the control of simplified purely *phenomenological* plant models using algebraic estimation techniques [8,9]. The idea of a controller with the capacity of lumped compensation of endogenous and exogenous disturbances by means of an observer based control was proposed by Han, introducing the concept of Active Disturbance Rejection Control (ADRC) [15]. ADRC controllers have led to a new paradigmatic view of traditional nonlinear control problems where disturbances, internal and external, are actively estimated and rejected. Experimental results have been reported in diverse examples of systems (see [6,13,14,36,38]). As mentioned in [23], linearized observer based control of nonlinear systems has produced successful implementations in disturbance canceling schemes. For the case of underactuated dynamical systems, observer based ADRC plus linearized flatness takes one further step into the ADRC control of nonlinearizable systems. Efforts on underactuated systems control have also been recently advanced with promising results (see [19,21,37]).

It has been shown that the use of approximate disturbance estimation, via extended Luenberger observers, known as Generalized Proportional Integral (GPI) observers [32], constitutes an effective manner of integrating ADRC schemes. The GPI observer naturally includes a self-updating, lumped, time-polynomial model of the nonlinear state-dependent perturbation (Ref. [18] studies another interesting approach of time-polynomial disturbance estimation technique). The GPI estimates the perturbations and delivers a time signal to the controller for on-line cancelation of the effect of unknown nonlinearities and foreign perturbations while, simultaneously, estimating the phase variables related to the measured output. The scheme achieves accurate on-line estimations of the joint effect of all unknown disturbances (state dependent or non state dependent). Some applications have been reported in the recent literature (see [30–33]). The main features of this control scheme lie in the fact that both, exogenous unstructured perturbation inputs and state-dependent perturbation inputs, appearing in the input–output model, are all lumped into a simplifying time-varying signal that needs to be linearly estimated. The control scheme takes advantage of the natural possibilities of differentially flat systems, allowing the use of linear disturbance estimation and linear output feedback control with disturbance cancelation (see [10] and the books [20,29] for a comprehensive treatment on differentially flat systems). The main difference between traditional flatness based controllers and the ADRC scheme for flat systems is the fact that traditionally flatness based controls need perfect knowledge of the plant while ARDC schemes for flat systems may largely ignore unknown nonlinearities and exogenous additive perturbation inputs in the input-to-flat output dynamics. Needless to say, flat systems do not have any zero dynamics, thus avoiding the problem of internal stability after feedback.

In this article, an ADRC scheme is proposed for a trajectory tracking problem associated with the Furuta pendulum. The tracking problem is that of having the horizontal arm follow a rest to rest angular position reference trajectory, while the unactuated pendulum is to remain around its unstable vertical position, without falling, during the entire tracking maneuver and long after it ceases. The control scheme assumes an important lack of knowledge of the system parameters, nonlinearities and exogenous disturbance signals. Using a tangent linearization model of the Furuta pendulum around an arbitrary equilibrium, we show that fast excursions from the unstable equilibrium point, triggering adverse effects of nonlinearities, are still feasible while maintaining the pendulum around its unstable vertical position. The scheme not only accurately estimates the effects of the excited nonlinearities, but it also reduces the tracking control problem to that defined on a chain of integrators after on-line active disturbance cancelations. The control scheme is tested on an experimental prototype, showing excellent results for the tracking error and the estimation of lumped state dependent and external disturbances.

Section 2 briefly considers the nonlinear model of the Furuta Pendulum and its tangent linearization around an arbitrary equilibrium point. In this section the flatness (controllability) of the linearized model is exploited to obtain a natural decoupled two stage observer design. The method is extendable to some other underactuated systems, i.e. the ball and beam, inverted pendulum on a cart, gantry crane systems, etc. Section 3 proposes a high gain extended linear observer based ADRC tracking scheme for the tangent linearized model of the system. High gain extended observers of the Luenberger type receive here the name of Generalized Proportional Integral (GPI) observers due to their dual relationship with robust linear GPI controllers, introduced by Fliess et al. [11]. Section 4 special emphasis is placed on a careful pole placement for the linear observer estimation dynamics, avoiding the traditional "peaking" phenomenon, appearing in high gain controlled systems. Section 5 is devoted to present the details of the experimental setup and discusses the obtained closed loop performance of the system; besides, the section illustrates the controller behavior by means of an experimental comparison against a sliding mode controller, under the same control task. The conclusions and suggestions for further research constitute the topic of the last section.

2. The Furuta pendulum

2.1. The nonlinear Furuta pendulum model

The Furuta pendulum system consisting of an unactuated pendulum attached to the end of a horizontal rotating arm (see Fig. 1). The pendulum is free to move on a plane perpendicular to the horizontal arm which is driven by a DC motor. The nonlinear

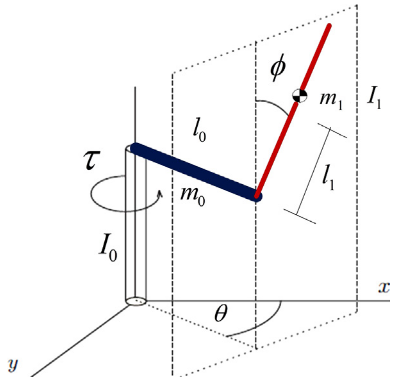

Fig. 1. Schematic of the Furuta Pendulum.

Please cite this article as: Ramírez-Neria M, et al. Linear active disturbance rejection control of underactuated systems: The case of the Furuta pendulum. ISA Transactions (2013), http://dx.doi.org/10.1016/j.isatra.2013.09.023