ISA Transactions 53 (2014) 947–954

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Limit cycle analysis of active disturbance rejection control system with two nonlinearities

Dan Wu, Ken Chen

State Key Laboratory of Tribology, Department of Mechanical Engineering, Tsinghua University, Beijing 100084, China

ARTICLE INFO

Article history:
Received 23 April 2013
Received in revised form
25 December 2013
Accepted 3 March 2014
Available online 1 May 2014
This paper was recommended for publication by Dr. Didier Theilliol

Keywords:
Nonlinear systems
Active disturbance rejection control
Limit cycle
Describing function

ABSTRACT

Introduction of nonlinearities to active disturbance rejection control algorithm might have high control efficiency in some situations, but makes the systems with complex nonlinearity. Limit cycle is a typical phenomenon that can be observed in the nonlinear systems, usually causing failure or danger of the systems. This paper approaches the problem of the existence of limit cycles of a second-order fast tool servo system using active disturbance rejection control algorithm with two *fal* nonlinearities. A frequency domain approach is presented by using describing function technique and transfer function representation to characterize the nonlinear system. The derivations of the describing functions for *fal* nonlinearities and treatment of two nonlinearities connected in series are given to facilitate the limit cycles analysis. The effects of the parameters of both the nonlinearity and the controller on the limit cycles are presented, indicating that the limit cycles caused by the nonlinearities can be easily suppressed if the parameters are chosen carefully. Simulations in the time domain are performed to assess the prediction accuracy based on the describing function.

1. Introduction

Many control problems involve uncertainties in the plant model parameters and external disturbances. The control design aims to deal with such uncertainties and disturbance by different strategies and algorithms in order to tolerate the uncertainties and minimize the negative effect of the disturbances on the system. Active disturbance rejection control (ADRC) is a novel robust control method that was systematically proposed by Han in his pioneer works [1,2]. In contrast to existing model-based designs, the ADRC does not need a precise analytical description of the system, as the unknown parts of dynamics are assumed as the internal disturbance in the plant, which, together with the external disturbance, is denoted as a generalized or total disturbance. Then, an extended state observer (ESO) that extends the system model with an additional and fictitious state variable is proposed to estimate the generalized disturbance and compensate for in the control law. This is a drastic departure because the information about the physical process needed by the controller is derived from the plant input–output data in real time and not from an a priori mathematical model. Therefore, the ADRC is more of a paradigm shift in feedback control system design than a new control design strategy [3].

On the other hand, instead of eliminating the presence of nonlinearity in most of the current control designs for simplicity, the ADRC intentionally introduces the nonlinearities into the design of the observer and the control law. Han has given insight into the nonlinear feedback mechanism and concluded that a nonlinear ADRC is potentially much more effective than a linear one and provides surprisingly better results in practice. A typical nonlinear ADRC framework is to develop the ESO with two nonlinear gains.

In fact, linear control methods rely on the key assumption of small range operation for the linear model to be valid. When the required operation range is large, a linear control controller is likely to perform very poorly or to be unstable, because the nonlinearities in the system cannot be compensated for. Nonlinear controllers may handle the nonlinearities in large range operation directly. Moreover, linear control may require high quality actuators and sensors to produce linear behavior in the specified operation range, while nonlinear control may permit the use of less expensive components with nonlinear characteristics. This will lower operation cost. As for performance optimality, bang–bang type controllers are one of favorite examples, which can produce fast response [4].

Due to active disturbance rejection concept and nonlinear feedback mechanism, the ADRC shows power and attraction to many control problems. This was originally demonstrated through time domain simulations and lately by various engineering applications, such as the trajectory tracking control of a flexible-joint robotic system [5], control for micro-electro-mechanical gyroscopes [6], speed control for permanent magnet synchronous motor servo system [7], and the control design for superconducting radio frequency cavities [8].

http://dx.doi.org/10.1016/j.isatra.2014.03.001
0019-0578/

Intentional nonlinearity is artificially introduced into the ADRC to make it more effective in tolerance to uncertainties and disturbance and improvement of system dynamics. In return, it may make the system produce some complex but colorful nonlinear behaviors, such as multiple equilibrium points, limit cycles, bifurcations and chaos. Specially, the limit cycle phenomenon is deserving of attention since it is apt to occur in any physical nonlinear system. A limit cycle can be desirable, for example, by providing the vibration that minimizes frictional effects in mechanical systems. On the other hand, a limit cycle can cause mechanical failure of a control system and other undesirable effects. Previous research paid little attention on the analysis of limit cycle behavior of nonlinear ADRC. This paper attempts to study limit cycle behavior of a nonlinear ADRC with application to a fast tool servo for non-rotationally symmetric turning. In such a machining process, the fast tool servo drives the cutting tool to move back and forth with given amplitude and frequency in order to track the desired motion trajectory [9]. The most typical application of the fast tool servo is ultra-precision machining of micro-structured surfaces, which is much sensitive to undesired limit cycle vibrations between the cutting tool and the workpiece. If limit cycles, or chatters occur during machining, it significantly affects the surface finish quality, causes loss of dimensional accuracy of the workpiece, and accelerates the premature wear, chipping, and failure of the cutting tool [10]. As a result, limit cycles are dangerous and should be avoided in the fast tool servo control design.

In order to study the nonlinear ADRC system, the describing function method is used to model the system in the frequency domain and characterize its limit cycles behavior in this paper. Describing function analysis is a widely known technique to study frequency response of nonlinear systems. It is an extension of linear frequency response analysis. In linear systems, transfer functions depend only on the frequency of the input signal. In nonlinear systems, when a specific class of input signal such as a sinusoidal wave is applied to a nonlinear element, one can represent the nonlinear element by a function that depend not only on the frequency, but also on the amplitude of the input signal [4]. The describing function method has been widely used to analyze nonlinear control problems such as predicting limit cycles [11–14], analyzing sliding mode observer dynamics [15], bifurcation analysis of nonlinear systems [16], and investigating the behavior of a fractional order Van der Pol-like oscillator [17].

Previous work [9] has studied the ADRC system with a single separable nonlinearity for simplification, focusing on analysis of stability, tracking and disturbance rejection performances of the system. However, the two-nonlinearity ADRC system might exhibit better control quality [1] but increase complexity of the system. Also, a great deal of the simplicity of the describing function approximate approach is lost when nonlinear effects are presented at more than one station in the system. Therefore, it is necessary to explore the performances of the ADRC system with multiple nonlinearities. Actually, it is quite usual that nonlinearities are presented at more than one station around a control loop. This demands to develop methodology of analyzing nonlinear behavior of the systems with multiple nonlinearities.

The rest of the paper is organized as follows. Section 2 gives a simple description of the fast tool servo and the ADRC design with two nonlinearities. Section 3 characterizes the two-nonlinearity ADRC in the frequency domain using both the describing function and the transfer function methods. A time-domain iteration algorithm is also presented to treat two nonlinearities to a single nonlinearity available for analysis of single frequency limit cycles. Section 4 discusses the effect of the controller parameters on the limit cycles. The time-domain simulation is also given to demonstrate the accuracy of limit cycle prediction based on the describing function method. The conclusions in Section 5 close the paper.

2. Design of the fast tool servo controller

In this section, a nonlinear active disturbance rejection controller is developed for the fast tool servo applied to ultra-precision machining of micro-structured surfaces.

2.1. Hardware of the fast tool servo

The fast tool servo is a typical precision, closed-loop linear motion control system. It consists of a normal-stress electromagnetically driven linear actuator [18], a power amplifier, a capacitance displacement sensor, a commercial Digital Signal Processing (DSP)-based motion control board and a self-developed interface board for implementation of the high speed A/D and D/A conversion as well as the I/O connection with the DSP. The whole system is controlled in real-time using an industrial computer with the host-target architecture [19]. The actuator and the power amplifier dynamics are considered to be the plant dynamics to be controlled. Fig. 1 illustrates the prototype of the developed electromagnetically driven fast tool servo.

A theoretical computation and off-line identification reveals that the plant dynamics can be approximated using a second-order LTI dynamic model in the form of

$$G_p(s) = \frac{b}{s^2 + ps + q}, \tag{1}$$

where the plant parameters are denoted as $p = 4.67 \times 10^3$/s, $q = 3.67 \times 10^6/\text{s}^2$ and $b_0 = b = 3 \times 10^7$ μm/s^2/V. The actuator travel is approximately 50 μm. The measurement resolution of the displacement sensor is 1 nm, and the maximum sampling control rate of the real-time computer is up to 500 kHz.

2.2. Design of nonlinear active disturbance rejection controller

Fast tool servo control is a typical precision motion control problem, where the control input must be applied so that the plant output follows desired constant or time varying signals. However, it is usual that there are different kinds of uncertainties in mechanical and electrical components, such as thrust force or torque constant of an actuator, nonlinear friction and stiffness, vibration mode frequencies. This challenges the controller

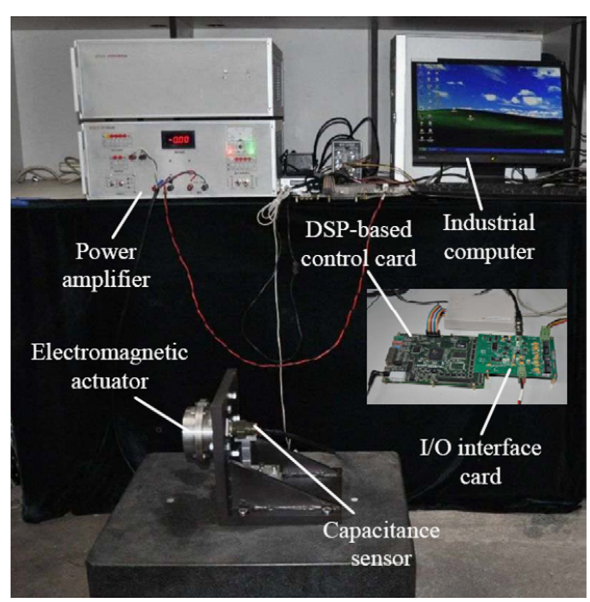

Fig.1. The picture of the electromagnetic fast tool servo case.