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Research Article

Input–output finite-time stabilization of linear systems with finite-time boundedness

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ABSTRACT

The paper presents linear system Input–Output Finite-Time Stabilization (IO-FTS) method under Finite-Time Boundedness (FTB) constraint. A state feedback controller is designed, via Linear Matrix Inequalities (LMIs), to guarantee the system both IO-FTS and FTB. The proposed methods are applied to the guidance design of a class of terminal guidance systems to suppress disturbances with IO-FTS method and FTB constraints simultaneously satisfied. The simulation results illustrate the effectiveness of the proposed methods.

1. Introduction

The concept of Finite-Time Stability (FTS) was introduced in 1960s. Up until now, much work has been done in this field. Given a bound on the initial condition, a system is said to be finite-time stable if the state does not exceed a certain threshold during a specified time interval. It worth noting that there is a different notion of FTS [1], which requires the system state to reach the system equilibrium in a finite time interval, and the property is called finite-time attractiveness in some research [2,3]. In the remainder of this paper, FTS we mentioned refers to the former one.

While external disturbances are considered, FTS is extended to Finite-Time Boundedness (FTB). In the light of results coming from Linear Matrix Inequality (LMI) theory, many optimization and control problems can be formulated and solved by using LMIs [4–8]. Sufficient conditions for FTB and finite-time stabilization of linear time-invariant system were provided [9]. Then the FTB problem was studied in many other cases, such as discrete-time linear systems [10], stabilization via dynamic output feedback [11], impulsive dynamical systems [12], linear time-varying systems with jumps [13] and impulsive dynamical linear systems [14]. Recently, necessary and sufficient conditions for finite-time stability of impulsive dynamical linear systems were also proposed [15].

Input–Output Finite-Time Stability (IO-FTS) has been given in [16], which means that, given a class of norm bounded input signals over a specified time interval $[0, T]$, the outputs of the system are also norm bounded over $[0, T]$. Amato et al. [16] provided methods to solve IO-FTS problem via static state feedback with the disturbance considered as input of the system. Therefore, this contribution can be used to disturbance suppression.

It can be concluded that FTB consider the system state (not exceeding a given threshold) in a finite time interval while IO-FTS only consider the output (norm bounded with input disturbances satisfying some boundedness conditions) in a finite time interval. However, in some kind of practical applications, system state boundedness and disturbance suppression in a finite time interval are both concerned. The FTB only concerns system state not exceeding a given threshold, and cannot take into account some other system performance measures. Hence, by combining FTB and IO-FTS, the behaviors of system state and output can be considered comprehensively, as dealt in this paper.

It should be mentioned that, while considering system behavior in INFINITE time, many disturbance suppression control methods can guarantee asymptotic stability of the systems [17], such as $\boldsymbol{H}_\infty$ Control and Linear Quadratic Optimal Control [18]. However, because of the independence of FTS and Lyapunov Asymptotic Stability (LAS) (A system can be FTS but not LAS,

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and vice visa. See [9]), finite-time disturbance suppression control methods always do not have a similar capability of stabilizing the system in the finite time interval.

The remainder of this paper is organized as follows. Section 2 presents the basic definitions and the problem statement. Section 3 proposes the sufficient conditions which guarantee both FTB and IO-FTS of a linear time-invariant system. State feedback controller design method is also proposed. Section 4 shows the application of the proposed methods to a class of terminal guidance systems. The conclusion is given in Section 5.

2. Problem statement

Consider the following time-invariant linear system

$$\dot{x}(t) = Ax(t) + Gw(t), \tag{1a}$$

$$y(t) = Cx(t), \tag{1b}$$

where $x(t) \in \mathbb{R}^n$ are the state, $w(t) \in \mathbb{R}^l$ are exogenous disturbance and satisfies

$$\int_0^T w^T(t) S_1 w(t) dt \le 1. \tag{2a}$$

Matrices $A \in \mathbb{R}^{n \times n}$, $G \in \mathbb{R}^{n \times l}$, $C \in \mathbb{R}^{m \times n}$ and $S_1 \in \mathbb{R}^{l \times l}$. S_1 is a positive definite symmetric matrix. We provide the following definitions, which is slightly different with the ones in [9] (FTB) and [16] (IO-FTS).

Definition 1 (FTB). (System (1) is said to be finite-time bounded (FTB) with respect to (c_1, c_2, T, R, S_1), with $c_1 < c_2$, $R > 0$, $T > 0$, if

$$x^T(0)Rx(0) \le c_1 \Rightarrow x^T(t)Rx(t) \le c_2, \ \forall t \in [0, T].$$

Definition 2 (IO-FTS). (Consider zero initial condition $(x(0) = 0)$, System (1) is said to be Input–Output Finite-Time Stable (IO-FTS) with respect to (T, S_2), with $T > 0$, $S_2 > 0$ if

$$y^T(t) S_2 y(t) \le 1, \ \forall t \in [0, T]. \tag{2b}$$

Remark 1. In the area of the finite time system area, most of scholars consider the following two classes of exogenous disturbances for the FTB and IO-FTS problems,

$$w(t) \in \mathrm{L}_{2,R}[0, T], \quad \int_0^T w^T(t) R w(t) dt \le 1$$

$$w(t) \in \mathrm{L}_{\infty,R}[0, T], \quad \max_{t \in [0,T]} w^T(t) R w(t) \le 1$$

For instance, Amato et al. [9] considers the FTB problem with the disturbance $w(t) \in \mathrm{L}_{\infty,R}[0, T]$ and Amato et al. [16] considers the IO-FTS with both two classes of disturbances. In this paper, we only focus on the first class of disturbance, i.e.

$$w(t) \in \mathrm{L}_{2,R}[0, T], \quad \int_0^T w^T(t) R w(t) dt \le 1 .$$

Problem 1 (Both FTB and IO-FTS via state feedback). Consider the system

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t), \tag{3a}$$

$$y(t) = Cx(t), \tag{3b}$$

where $u(t) \in \mathbb{R}^r$ is the input, $B \in \mathbb{R}^{n \times r}$. Consider a state feedback

$$u(t) = Kx(t) \tag{4}$$

where K is a matrix to be determined later. Applying this controller to the system (3) resulting the following closed-loop system:

$$\dot{x}(t) = \overline{A}x(t) + Gw(t), \tag{5a}$$

$$y(t) = Cx(t). \tag{5b}$$

where $\overline{A} = A + BK$. The problem is to find a state feedback controller in the form of (4) such that the closed-loop system (5) is FTB with respect to (c_1, c_2, T, R, S_1) and IO-FTS with respect to (T, S_2).

3. Main result

Lemma 1 (sufficient condition of FTB). *System (1) is FTB with respect to (c_1, c_2, T, R, S_1) if, letting $\tilde{Q} = R^{-\frac{1}{2}} Q R^{-\frac{1}{2}}$, there exit a scalar $\alpha > 0$ and a symmetric positive definite matrix $Q \in \mathbb{R}^{n \times n}$ such that*

$$\begin{bmatrix} A\tilde{Q} + \tilde{Q}A^T - \alpha\tilde{Q} & G \\ G^T & -S_1 \end{bmatrix} < 0, \tag{6a}$$

$$1 + \frac{c_1}{\lambda_{\min}(Q)} < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(Q)}, \tag{6b}$$

where $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ indicate the maximum and minimum eigenvalues of argument, respectively.

Proof. Let $V(x(t)) = x^T(t)\tilde{Q}^{-1}x(t)$, we have

$$\dot{V} = \begin{bmatrix} x \\ w \end{bmatrix}^T \begin{bmatrix} A^T\tilde{Q}^{-1} + \tilde{Q}^{-1}A & \tilde{Q}^{-1}G \\ G^T\tilde{Q}^{-1} & 0 \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix}. \tag{7}$$

We omit (t) whenever no ambiguity arises.

Pre and post-multiplying (6a) by

$$\begin{bmatrix} \tilde{Q}^{-1} & 0 \\ 0 & I \end{bmatrix},$$

we obtain

$$\begin{bmatrix} A^T\tilde{Q}^{-1} + \tilde{Q}^{-1}A - \alpha\tilde{Q}^{-1} & \tilde{Q}^{-1}G \\ G^T\tilde{Q}^{-1} & -S_1 \end{bmatrix} < 0. \tag{8}$$

Putting together (7) and (8), we have

$$\dot{V} - \alpha V - w^T S_1 w < 0. \tag{9}$$

Pre and post-multiplying (9) by $e^{-\alpha t}$, and integrating from 0 to t, $t \in (0, T]$, we obtain

$$\int_0^t (e^{-\alpha\tau}V)' d\tau < \int_0^t e^{-\alpha\tau} w^T S_1 w d\tau.$$

Noting $\alpha > 0$, we have

$$e^{-at}V(x(t)) - V(x(0)) < \int_0^t w^T S_1 w d\tau \le 1,$$

then

$$V(x(t)) < e^{at}(1 + V(x(0))),$$

which can be rewritten as

$$x^T(t)R^{1/2}Q^{-1}R^{1/2}x(t) < e^{at}(1 + x^T(0)R^{1/2}Q^{-1}R^{1/2}x(0)),$$

then

$$\lambda_{\min}(Q^{-1})x^T(t)Rx(t) < e^{at}(1 + \lambda_{\max}(Q^{-1})x^T(0)Rx(0)). \tag{10}$$

By (6b) and (10), we can obtain, for all $t \in [0, T]$

$$x^T(t)Rx(t) \le c_2$$

Therefore, the proof follows.