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Research Article

New delay dependent stability criteria for recurrent neural networks with interval time-varying delay ☆

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ABSTRACT

This paper is concerned with the delay dependent stability criteria for a class of static recurrent neural networks with interval time-varying delay. By choosing an appropriate Lyapunov–Krasovskii functional and employing a delay partitioning method, the less conservative condition is obtained. Furthermore, the LMIs-based condition depend on the lower and upper bounds of time delay. Finally, a numerical example is also designated to verify the reduced conservatism of developed criteria.

1. Introduction

Over the past two decades, recurrent neural networks have attracted considerable attention in various fields such as handwriting recognition, automatic control and chemical processes [1–4]. However, time delay in the practical systems always causes the poor performance and instability. Furthermore, because the lower bound is not always restricted to zero, time delay may be time-varying, and the systems are referred to recurrent neural networks with time-varying delays. Therefore, the issue of stability of recurrent neural networks with time-varying delay is of great important. Up to now, many results on stability analysis of recurrent neural networks have been obtained [5–18].

It is well known that the method of Lyapunov–Krasovskii functional can make full use of time delay. It should be noted that most of the obtained results are always classified into the delay-independent criteria and delay-dependent one. Compared with stability criteria subject to delay-independent, the delay-dependent ones usually received less conservatism and has become the main issue of primary significance. In order to derive the less conservative condition, many techniques have been employed in terms of linear matrix inequalities (LMIs), such as free weighting matrix, delay decomposition, and Jensen's integral inequalities [10–24]. However, there still exists some conservatism for recurrent neural networks with interval time-varying delay waiting for the further improvements. It is very important to choose a new technique with Lyapunov–Krasovskii functional and obtain an upper bound of time delay.

This paper focuses on the delay-dependent stability criteria for recurrent neural networks with interval time-varying delays. The main contribution of this paper lies in the fact that the integral inequalities are employed, both lower and upper bounds of time delays are also taken into consideration when developing a new Lyapunov–Krasovskii functional, the less conservative stability result is obtained. It should be mentioned that, regarding the previous convex method which only tackles the time-varying delay, our techniques apply the idea of combined convex technique to both delay and delay variation. At last, a numerical example is given to demonstrate the reduced conservatism and the effectiveness of the proposed method.

2. Preliminaries

Consider a class of static recurrent neural network with time-varying delays described as follows:

$$\dot{x}(t) = -Ax(t) + g(Wx(t-\tau(t)) + J), \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^\top$ denotes the neuron state vector, $A = \mathrm{diag}\{a_1, a_2, \dots, a_n\}$ with $a_i > 0$, $i = 1, 2, \dots, n$, $g(Wx(\cdot)) = [g_1(W_1x(\cdot)), g_2(W_2x(\cdot)), \dots, g_n(W_nx(\cdot))]^\top$ is the neuron activation function. $W = [W_1^\top, W_2^\top, \dots, W_n^\top]]^\top$ is the delayed connection weight matrix. $J = [j_1, j_2, \dots, j_n]^\top$ is a constant input. $\tau(t)$ is the

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time-varying delay and satisfies

$$\tau_0 \le \tau(t) \le \tau_M, \tag{2}$$

$$d_0 \le \dot{\tau}(t) \le d_M. \tag{3}$$

where $d_0 \le d_M$ and $0 < \tau_0 \le \tau_M$ are the constant scalars.

Assumption 1. Each bounded neuron activation function, $g_i(\cdot)$, $i = 1, 2, \ldots, n$, satisfies

$$l_i^- \le \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \le l_i^+, \quad \forall s_1 \ne s_2, \ i = 1, 2, \ldots, n, \tag{4}$$

where l_i^+, l_i^-, $i = 1, 2, \ldots, n$, are known real constants.

Assumption 1 guarantees the existence of an equilibrium point of system (1). Denote that $x^* = [x_1^*, x_2^*, \ldots, x_n^*]$ is the equilibrium point. Using the transformation $\eta(\cdot) = x(\cdot) - x^*$, system (1) can be converted to the following error system:

$$\dot{\eta}(t) = -A\eta(t) + f(Wx(\eta - \tau(t))), \tag{5}$$

where $\eta(t) = [\eta_1(t), \eta_2(t), \ldots, \eta_n(t)]^\top$ is the state vector, $f(Wx(\cdot)) = [f_1(W_1\eta(\cdot)), f_2(W_2\eta(\cdot)), \ldots, f_n(W_n\eta(\cdot))]^\top$ with $f(W\eta(\cdot)) = g(W(\eta(\cdot) + x^*) + J) - g(Wx^* + J)$. It is easy to see that $f_i(\cdot)$, $i = 1, 2, \ldots, n$, satisfies

$$b_i \le \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \le l_i, \quad \forall s_1 \ne s_2, \ i = 1, 2, \ldots, n. \tag{6}$$

The following integral inequalities are introduced in the following lemma which is important in the derivation of main results.

Lemma 2.1 (*Zhang et al.* [5]). *Let $\eta(t) \in \mathbb{R}^n$ be a vector-valued function with first-order continuous-derivative entries. Then, for any matrices $M \in \mathbb{R}^{n\times n}$, $N \in \mathbb{R}^{n\times n}$, $Z \in \mathbb{R}^{2n\times 2n}$, $R = R^\top \in \mathbb{R}^{n\times n}$, and some given scalars $0 \le \tau_0 < \tau_M$, the following integral inequality holds:*

(1) *When τ_0, τ_M are time-varying, $h = \tau_M - \tau_0$, and R is any symmetric matrix,*

$$-\int_{t-\tau_M}^{t-\tau_0} \dot{\eta}^\top(s) R \dot{\eta}(s)\, ds \le \begin{bmatrix} \eta(t-\tau_0) \\ \eta(t-\tau_M) \end{bmatrix}^\top \left\{ \begin{bmatrix} M + M^\top & -M + N^\top \\ * & -N - N^\top \end{bmatrix} + hZ \right\} \begin{bmatrix} \eta(t-\tau_0) \\ \eta(t-\tau_M) \end{bmatrix} \tag{7}$$

with $\begin{bmatrix} R & Y \\ * & Z \end{bmatrix} \ge 0$ and $Y = [M\ N]$.

(2) *When $R > 0$ and τ_0, τ_M are time-varying, $h = \tau_M - \tau_0$*

$$-\int_{t-\tau_M}^{t-\tau_0} \dot{\eta}^\top(s) R \dot{\eta}(s)\, ds \le \begin{bmatrix} \eta(t-\tau_0) \\ \eta(t-\tau_M) \end{bmatrix}^\top \left\{ \begin{bmatrix} M + M^\top & -M + N^\top \\ * & -N - N^\top \end{bmatrix} + h \begin{bmatrix} M \\ N \end{bmatrix} R^{-1} [M^\top \ \ N^\top] \right\} \begin{bmatrix} \eta(t-\tau_0) \\ \eta(t-\tau_M) \end{bmatrix} \tag{8}$$

Lemma 2.2 (*Yan et al.* [6]). *For any vectors a, b and any positive definite symmetric matrices P, then*

$$\pm 2a^\top b \le a^\top P^{-1} a + b^\top P b.$$

Lemma 2.3. *Suppose $\tau_0 \le \tau(t) \le \tau_M$, for any symmetric positive definite matrix R and matrices T, Y, the following inequality holds:*

$$\begin{aligned} -\int_{t-\tau_M}^{t-\tau_0} \eta^\top(s) R \eta(s)\, ds \le &\left[\int_{t-\alpha\tau(t)}^{t-\tau_0} \eta^\top(s) R \eta(s)\, ds \quad \int_{t-\tau(t)}^{t-\alpha\tau(t)} \eta^\top(s) R \eta(s)\, ds \quad \int_{t-\tau_M}^{t-\tau(t)} \eta^\top(s) R \eta(s)\, ds \right] \\ &\times \Big\{ (\alpha\tau(t) - \tau_0) X R^{-1} X + (1-\alpha)\tau(t) Y R^{-1} Y \\ &+ (\tau_M - \tau(t)) Z R^{-1} Z^\top + [X\ Y\ Z] + [X\ Y\ Z]^\top \Big\} \\ &\times \left[\int_{t-\alpha\tau(t)}^{t-\tau_0} \eta^\top(s) R \eta(s)\, ds \quad \int_{t-\tau(t)}^{t-\alpha\tau(t)} \eta^\top(s) R \eta(s)\, ds \quad \int_{t-\tau_M}^{t-\tau(t)} \eta^\top(s) R \eta(s)\, ds \right]^\top \end{aligned} \tag{9}$$

Proof. Denoting $\Theta(t) = [\int_{t-\tau_0}^{t-\alpha\tau(t)} \eta^\top(s) R \eta(s)\, ds \ \int_{t-\tau(t)}^{t} \ -\alpha\tau(t)\eta^\top(s) R\eta(s)\, ds \ \int_{t-\tau_M}^{t-\tau(t)} \eta^\top(s) R \eta(s)\, ds]^\top$, $X = [X_1^\top\ X_2^\top\ X_3^\top]^\top$, $Y = [Y_1^\top\ Y_2^\top\ Y_3^\top]^\top$, $Z = [Z_1^\top\ Z_2^\top\ Z_3^\top]^\top$, using Lemma 2.2, one has

$$\begin{aligned} &-2\Theta^\top(t) X \int_{t-\alpha\tau(t)}^{t-\tau_0} \eta(s)\, ds \le (\alpha\tau(t) - \tau_0)\Theta^\top(t) X R^{-1} X^\top \Theta(t) \\ &+ \frac{1}{\alpha\tau(t) - \tau_0} \left[\int_{t-\alpha\tau(t)}^{t-\tau_0} \eta(s)\, ds \right]^\top R \left[\int_{t-\alpha\tau(t)}^{t-\tau_0} \eta(s)\, ds \right] \\ &\le (\alpha\tau(t) - \tau_0)\Theta^\top(t) X R^{-1} X^\top \Theta(t) + \int_{t-\alpha\tau(t)}^{t-\tau_0} \eta^\top(s) R \eta(s)\, ds. \end{aligned}$$

which can be rewritten as follows:

$$\begin{aligned} -\int_{t-\alpha\tau(t)}^{t-\tau_0} \eta^\top(s) R \eta(s)\, ds \le \Theta^\top(t) \{ &(\alpha\tau(t) - \tau_0) X R^{-1} X^\top \\ &+ [X\ 0\ 0] + [X\ 0\ 0]^\top \} \Theta(t). \end{aligned}$$

Similarly, one also has

$$\begin{aligned} -\int_{t-\tau(t)}^{t-\alpha\tau(t)} \eta^\top(s) R \eta(s)\, ds \le \Theta^\top(t) \{ &(1-\alpha)\tau(t) Y R^{-1} Y^\top \\ &+ [0\ Y\ 0] + [0\ Y\ 0]^\top \} \Theta(t), \end{aligned}$$

$$\begin{aligned} -\int_{t-\tau_M}^{t-\tau(t)} \eta^\top(s) R \eta(s)\, ds \le \Theta^\top(t) \{ &(\tau_M - \tau(t)) Z R^{-1} Z^\top \\ &+ [0\ 0\ Z] + [0\ 0\ Z]^\top \} \Theta(t). \end{aligned}$$

In the end, a summation of the last three equations completes the proof. □

3. Main results

In this section, we aim to analyze the asymptotic stability for recurrent neural network with time-varying delays.

For convenience, denote

$$\begin{aligned} \xi^\top(t) = \mathrm{col}\Big\{ &\eta^\top(t)\ \eta^\top(t-\alpha\tau(t))\ \eta^\top(t-\tau(t))\ \eta^\top(t-\tau_0)\ \eta^\top(t-\tau_M)\ \dot{\eta}^\top(t-\alpha\tau(t))\ \dot{\eta}^\top(t-\tau(t))\ \dot{\eta}^\top(t-\tau_0) \dot{\eta}^\top(t-\tau_M)\ f^\top(W\eta(t))\ f^\top(W\eta(t-\alpha\tau(t)))\ f^\top(W\eta(t-\tau(t)))\ f^\top(W\eta(t-\tau_0))\ f^\top(W\eta(t-\tau_M)) \\ &\int_{t-\tau_0}^{t-\alpha\tau(t)} \eta^\top(s) R \eta(s)\, ds \ \int_{t-\tau(t)}^{t-\alpha\tau(t)} \eta^\top(s) R \eta(s)\, ds \ \int_{t-\tau_M}^{t-\tau(t)} \eta^\top(s) R \eta(s)\, ds \Big\}, \end{aligned}$$

$$e_s = \{\underbrace{0, \ldots, 0}_{s-1}, I, \underbrace{0, \ldots, 0}_{17-s}\}.$$

Theorem 3.1. *Given the constant scalars $\tau_M > \tau_0 \ge 0$, $d_M \ge d_0$, $1 > \alpha > 0$, the neural networks (1) are asymptotically stable if there exist mode-dependent symmetric matrices $\mathcal{P} > 0$, $Q_i > 0$, $R_i > 0$ $i = 1, \ldots, 4$, $S > 0$, Λ_j, Δ_j, M_j, N_j, $j = 1, 2, \ldots, 5$, $V > 0$, K_l, $l = 1, 2, 3$, such that the following matrix inequalities hold:*

$$\Xi(\tau(t), \dot{\tau}(t)) < 0, \tag{10}$$

$$\begin{bmatrix} \dot{\tau}(t) R_4 + (1 - \dot{\tau}(t)) R_3 & [M_2\ N_2] \\ * & Z_1 \end{bmatrix} \ge 0, \tag{11}$$