ARTICLE IN PRESS

ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Improved delay-dependent stability analysis for neural networks with time-varying delays

Xiangjun Xie [1], Zerong Ren *,[1]

School of Sciences, Southwest Petroleum University, Chengdu, Sichuan 610500, China

ARTICLE INFO

Article history:
Received 10 October 2013
Received in revised form
2 December 2013
Accepted 10 May 2014
This paper was recommended for publication by Dr. Q.-G. Wang.

Keywords:
Delay-dependent stability
Neural networks
Time-varying delays
Linear matrix inequality (LMI)

ABSTRACT

In this paper, the problem of delay-dependent asymptotic stability analysis for neural networks with time-varying delays is considered. A new class of Lyapunov functional is proposed by considering the information of neuron activation functions adequately. By using the delay-partitioning method and the reciprocally convex technique, some less conservative stability criteria are obtained in terms of linear matrix inequalities (LMIs). Finally, two numerical examples are given to illustrate the effectiveness of the derived method.

1. Introduction

In the past few decades, neural networks has been widely applied to various scientific problems, such as pattern recognition, image processing, and associative memories. Meanwhile, since time delay is inevitably encountered in neural networks, which are the main source of instability and oscillation, much attention has been paid to delayed neural networks and many elegant stability results have been proposed. So far, the stability criteria of delayed neural networks are classified into two categories, i.e., delay-independent [1–3], and delay-dependent [4–28]. Since delay-dependent stability criteria are usually less conservative than delay-independent ones, especially when the time delay is relatively small, much effort has been paid to the delay-dependent category. For the delay-dependent case, many stability criteria have been obtained by choosing the Lyapunov–Krasovskii functional (LKF). It is well known that the choice of an appropriate LKF is crucial for obtaining less conservative stability results. Thus, many new improved methods have been developed for reducing conservatism in recent years, such as free-weighting matrix [5–7], augmented LKF [8], and delay-slope-dependent method [9]. Obviously, it is hard to reduce the conservatism further by employing the identical LKFs. Therefore, the delay-partitioning method has been developed to investigate the stability of delayed neural networks, which significantly reduce the conservatism of the obtained stability criteria [11–19]. For example, in [11], by utilizing different free-weighting matrices in each delay subinterval, some improved stability criteria were proposed to reduce the conservatism for neural networks with time-varying delays. However, the delay-partition number has been chosen as two in [11–13]. When the number of subintervals becomes larger, the results will be less conservative. Very recently, in [14–19],a generalized delay-partitioning method was proposed to reduce the conservatism for delayed neural networks. For example, in [14], by dividing the delay interval $[0,h]$ into some subintervals with the same size, a new stability result for delayed Hopfield neural networks was proposed. In [15], the weighting-delay-based stability criteria for neural networks with time-varying delay were derived by dividing the delay interval with the weighted parameters. The derived results proved to be less conservative than most of the previous results, and the conservatism can be notably reduced by increasing the subintervals. However, these methods suffer from two common shortcomings. First, when the delay is time-varying, the relationship between time-varying delay and each subinterval is completely neglected. Second, the information of neuron activation functions is not adequately considered, which may lead to much conservatism. Thus, there is room for further investigation.

In this paper, for any integer $m \geq 1$, let $h = d/m$, then the delay interval $[0,d]$ can be decomposed into m segments, i.e., $[0,d] = \cup_{i=1}^{m} [(i-1)h, ih]$. Some existing results [14–18] only consider the information $\tau(t) \in [0,d]$, and the important information $\tau(t) \in [(k-1)h, kh], k = 1,2,\ldots,m$ is ignored, which may lead to considerable conservatism. Thus, when handling the time

* Corresponding author.
E-mail addresses: xiangjunxie1@163.com (X. Xie), zerongren@163.com (Z. Ren).
[1] Tel./fax: +86 28 83037655.

http://dx.doi.org/10.1016/j.isatra.2014.05.010
0019-0578/

derivative of $V(z_t)$, not only the information $\tau(t) \in [0, d]$ but also the important information $\tau(t) \in [(k-1)h, kh], k = 1, 2, \ldots, m$ is considered. So the relationship between time-varying delay $\tau(t)$ and each subinterval $[(k-1)h, kh], k = 1, 2, \ldots, m$ is considered adequately, which may lead to less conservative results. By taking more information of neuron activation functions, an augmented LKF is introduced to derive the stability results for the studied system. Then, inspired by the results of [30], a less conservative result is obtained to guarantee the asymptotically stable neural networks with time-varying delay. Finally, two numerical examples are given to indicate significant improvements over most of the existing results.

2. Problem formulation

Consider the following neural networks with time-varying delay:

$$\dot{x}(t) = -Cx(t) + Ag(x(t)) + Bg(x(t-\tau(t))) + \mu \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathcal{R}^n$ is the neuron state vector, $g(x(\cdot)) = [g_1(x_1(\cdot)), g_2(x_2(\cdot)), \ldots, g_n(x_n(\cdot))]^T \in \mathcal{R}^n$ denotes the neuron activation function, $\mu = (\mu_1, \mu_2, \ldots, \mu_n)^T \in \mathcal{R}^n$ is a constant input vector. $A \in \mathcal{R}^{n\times n}$ is the connection weight matrix and $B \in \mathcal{R}^{n\times n}$ is the delayed connection weight matrix. $C = \mathrm{diag}(C_1, C_2, \ldots, C_n)$ is a diagonal matrix with $C_i > 0$, $i = 1, 2, \ldots, n$. $\tau(t)$ is time-varying continuous function that satisfies $0 \le \tau(t) \le d$, $\dot{\tau}(t) \le u$, where d and u are constants. In addition, it is assumed that each neuron activation function $g_i(\cdot), i = 1, 2, \ldots, n$, satisfies the following condition:

$$k_i^- \le \frac{g_i(x) - g_i(y)}{x - y} \le k_i^+, \quad \forall x, y \in R,\ x \ne y,\ i = 1, 2, \ldots, n \tag{2}$$

where $k_i^-, k_i^+, i = 1, 2, \ldots, n$ are constants.

Assuming that $x^* = [x_1^*, x_2^*, \ldots, x_n^*]^T$ is the equilibrium point of system (1) whose uniqueness has been proven in [20]. By the transformation $z(\cdot) = x(\cdot) - x^*$, system (1) can be converted to the following form:

$$\dot{z}(t) = -Cz(t) + Af(z(t)) + Bf(z(t-\tau(t))) \tag{3}$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T$, $f(z(\cdot)) = [f_1(z_1(\cdot)), f_2(z_2(\cdot)), \ldots, f_n(z_n(\cdot))]^T$ and $f_i(z_i(\cdot)) = g_i(z_i(\cdot) + x_i^*) - g_i(x_i^*), i = 1, 2, \ldots, n$. According to inequality (2), we obtain

$$k_i^- \le \frac{f_i(z_i(t))}{z_i(t)} \le k_i^+ f_i(0) = 0, \quad i = 1, 2, \ldots, n \tag{4}$$

Thus, under above assumption, the following inequality is true for any positive diagonal matrix R, such that

$$z^T(t)KRKz(t) - f^T(z(t))Rf(z(t)) \ge 0 \tag{5}$$

where $K = \mathrm{diag}(k_1, k_2, \ldots, k_n)$, $k_i = \max(|k_i^-|, |k_i^+|)$

Lemma 1 *(See Zhu and Wang [21]). The following inequalities are true:*

$$0 \le \int_0^{z_i(t)} (k_i^+ s - f_i(s))\, ds \le (k_i^+ z_i(t) - f_i(z_i(t)))z_i(t) \tag{6}$$

Lemma 2 *(See Boyd et al. [29]). For any constant matrix $Z \in \mathcal{R}^{n\times n}$, $Z = Z^T > 0$, scalars $h > 0$, then*

$$-h\int_{t-h}^{t} x^T(s)Zx(s)\, ds \le -\int_{t-h}^{t} x^T(s)\, dsZ\int_{t-h}^{t} x(s)\, ds \tag{7}$$

Lemma 3 *(Park et al. [30]). Let $f_1, f_2, \ldots, f_N : \mathcal{R}^m \to \mathcal{R}$ have positive values in an open subset D of $\mathcal{R}^m$. Then, the reciprocally convex combination of f_i over D satisfies*

$$\min_{\{\alpha_i | \alpha_i > 0, \sum_i \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \ne j} g_{i,j}(t) \tag{8}$$

subject to

$$\left\{ g_{i,j} : R^m \to R, g_{j,i}(t) \triangleq g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \ge 0 \right\} \tag{9}$$

3. Main results

Theorem 1. *Given scalars $h \ge 0, u$, diagonal matrices $K_1 = \mathrm{diag}(k_1^-, k_2^-, \ldots, k_n^-)$, $K_2 = \mathrm{diag}(k_1^+, k_2^+, \ldots, k_n^+)$, for positive integer $m \ge 1$, $h = d/m$, the system (3) is globally asymptotically stable if there exist symmetric positive matrices $X = [X_{ij}]_{m\times m}$, $Q = [Q_{ij}]_{2\times 2}$, $R_i(i = 1, 2, \ldots, m)$, positive diagonal matrices $T_1, T_2, R, \Delta = \mathrm{diag}(\delta_1, \delta_2, \ldots, \delta_n)$, $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$, $T = \mathrm{diag}(t_1, t_2, \ldots, t_n)$, and any matrices $S_i(i = 1, 2, \ldots, m)$ with appropriate dimensions, such that for $k = 1, 2, \ldots, m$*

$$\Omega^{(k)} = \begin{bmatrix} \Omega_{11}^{(k)} & \mathcal{A}^T\hat{R} \\ * & -\hat{R} \end{bmatrix} < 0 \tag{10}$$

$$\Psi^{(k)} = \begin{bmatrix} R_k & S_k \\ * & R_k \end{bmatrix} > 0 \tag{11}$$

where

$$\Omega_{11}^{(k)} = \Sigma + \Phi_{11} + \overline{\Phi}_{11}^{(k)} + \tilde{\Phi}_{11}^{(k)}$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & X_{12} & \cdots & X_{1m} & 0 & 0 & \Sigma_{1(m+3)} & \Sigma_{1(m+4)} \\ * & X_{22} - X_{11} & \cdots & X_{2m} - X_{1(m-1)} & -X_{1m} & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & \cdots & X_{mm} - X_{(m-1)(m-1)} & -X_{(m-1)m} & 0 & 0 & 0 \\ * & * & \cdots & * & -X_{mm} & 0 & 0 & 0 \\ * & * & \cdots & * & * & \Sigma_{(m+2)(m+2)} & 0 & \Sigma_{(m+2)(m+4)} \\ * & * & \cdots & * & * & * & \Sigma_{(m+3)(m+3)} & (\Lambda - \Delta)B \\ * & * & \cdots & * & * & * & * & \Sigma_{(m+4)(m+4)} \end{bmatrix}$$

$$\Phi_{11} = \begin{bmatrix} \varphi_1 & R_1 & \cdots & 0 & 0 & 0 & 0 & 0 \\ * & \varphi_2 & \cdots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & \cdots & \varphi_m & R_m & 0 & 0 & 0 \\ * & * & \cdots & * & \varphi_{m+1} & 0 & 0 & 0 \\ * & * & \cdots & * & * & 0 & 0 & 0 \\ * & * & \cdots & * & * & * & 0 & 0 \\ * & * & \cdots & * & * & * & * & 0 \end{bmatrix}$$

Please cite this article as: Xie X, Ren Z. Improved delay-dependent stability analysis for neural networks with time-varying delays. ISA Transactions (2014), http://dx.doi.org/10.1016/j.isatra.2014.05.010