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Research Article

Robust synchronization of an array of neural networks with hybrid coupling and mixed time delays

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ABSTRACT

This paper is concerned with the global exponential synchronization for an array of hybrid coupled neural networks with leakage delay, time-varying discrete and distributed delays. By employing a novel augmented Lyapunov–Krasovskii functional (LKF), applying the theory of Kronecker product of matrices, Barbalat's Lemma and the technique of linear matrix inequality (LMI), delay-dependent sufficient conditions are obtained for the global exponential synchronization of the system. As an extension, robust synchronization criteria are derived for the corresponding system with parameter uncertainties. Some examples are given to show the effectiveness of the obtained theoretical results.

1. Introduction

Since small-world and scale-free complex networks were proposed in [1,2], complex dynamical networks, which are a set of interconnected nodes with specific dynamics, have received increasing attention from various fields of science and engineering such as the World Wide Web, electrical power grids, communication networks, the Internet, and global economic markets. Many interesting behaviors have been observed from complex dynamical networks, e.g., synchronization, consensus, self-organization, and spatiotemporal chaos spiral waves. Synchronization as an important collective behavior of complex dynamical networks has been widely investigated in the last two decades (see, for example, [3–7]).

Coupled neural networks (CNNs), as a special kind of complex networks, have been found to exhibit more complicated and unpredictable behaviors than a single neural network. Particularly, synchronization in an array of coupled neural networks, which is one of the hot research fields of complex networks, has been a challenging issue due to its potential applications in many areas such as secure communication, information science, chaos generator design, and harmonic oscillation generation. On the other hand, in the applications of neural networks, there exist unavoidably time delays due to the finite information processing speed and the finite switching speed of amplifiers. It is well known that time delay can cause oscillation and instability of neural networks. Therefore, various synchronization criteria for CNNs with time delays have been investigated in the literature [8–21] and references therein. To mention a few representative works, the synchronization problems for an array of neural networks with hybrid coupling and interval time-varying delay were investigated in [8]. In [9], the global exponential synchronization was investigated for an array of asymmetric neural networks with time-varying delays and nonlinear coupling. Cao and Li [10] presented cluster synchronization criteria for an array of hybrid coupled neural networks with delay. Park et al. [11] obtained the delay-dependent synchronization conditions for coupled discrete-time neural networks with interval time-varying delays in network coupling.

So far, very little attention has been paid to neural networks with time delay in a leakage (or “forgetting”) term (see [22–26]). This is due to some theoretical and technical difficulties. In fact, time delay in the leakage term also has great impact on the dynamics of neural networks. As pointed out by Gopalsamy [27], time delay in the stabilizing negative feedback term has a tendency to destabilize a system. On the other hand, it has been observed that neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths. It is desired to model them by introducing continuously distributed delays over a certain duration of time such that the distant past has less influence compared to the recent behavior of the state (see [16,17]). However, to the best of our knowledge, there are no global exponential synchronization

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results about an array of hybrid coupled neural networks with leakage delay, time-varying discrete and distributed delays.

In the real world, there exist inevitably some uncertainties caused by the existence of modeling errors, external disturbances, and parameter fluctuations, which would lead to complex dynamic behaviors (see [28–31]). Thus, a good neural network should have robustness against such uncertainties. Recently, robust synchronization problems against such uncertainties of complex neural networks were investigated in [32–34] and references therein.

Motivated by the works of Zhang et al. [8], Wang et al. [17], Park et al. [22] and the discussions above, the purpose of this paper is to present some new sufficient conditions for the robust global exponential synchronization of hybrid coupled neural networks with leakage delay, time-varying delays and parameter uncertainties. To this end, we consider the following differential equation system:

$$\begin{aligned}\dot{x}_i(t) &= -\hat{A}x_i(t-\tau)+\hat{W}_1 f(x_i(t))+\hat{W}_2 f(x_i(t-h(t)))\\ &\quad +\hat{W}_3\int_{t-\sigma(t)}^{t} f(x_i(s))\,ds+u(t)+\sum_{j=1}^{N} g_{ij}^{(1)}\hat{D}_1 x_j(t)\\ &\quad +\sum_{j=1}^{N} g_{ij}^{(2)}\hat{D}_2 x_j(t-h(t))+\sum_{j=1}^{N} g_{ij}^{(3)}\hat{D}_3\int_{t-\sigma(t)}^{t} x_j(s)\,ds, \qquad (1.1)\end{aligned}$$

where $i=1,2,\ldots,N$, N is the number of coupled nodes, $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^{\mathrm{T}}\in R^n$ is the neuron state vector of the ith node, n denotes the number of neurons in a neural network, $f(x_i(\cdot))=(f_1(x_{i1}(\cdot)),f_2(x_{i2}(\cdot)),\ldots,f_n(x_{in}(\cdot)))^{\mathrm{T}}\in R^n$ and $u(t)=(u_1(t),u_2(t),\ldots,u_n(t))^{\mathrm{T}}\in R^n$ denote the neuron activation function vector and the external input vector, respectively, τ, $h(t)$ and $\sigma(t)$ denote the leakage delay, time-varying discrete delay and distributed delay, respectively, $\hat{A}=A+\Delta A(t)$, $\hat{W}_k=W_k+\Delta W_k(t)$, $\hat{D}_k=D_k+\Delta D_k(t)$ $(k=1,2,3)$, $A=\mathrm{diag}\{a_1,a_2,\ldots,a_n\}\in R^{n\times n}$ $(a_k>0,\ k=1,2,\ldots,n)$ is the self-feedback matrix, $W_k\in R^{n\times n}$ $(k=1,2,3)$ are the connection weight matrices, $D_k\in R^{n\times n}$ $(k=1,2,3)$ are the constant inner-coupling matrices of nodes, which describe the individual coupling between networks, $\Delta A(t)$, $\Delta W_k(t)$ and $\Delta D_k(t)$ represent the parameter uncertainties of the system, which are assumed to be of the form

$$\begin{aligned}&[\Delta A(t)\ \ \Delta W_1(t)\ \ \Delta W_2(t)\ \ \Delta W_3(t)\ \ \Delta D_1(t)\ \ \Delta D_2(t)\ \ \Delta D_3(t)]\\ &\quad = M\nabla(t)[E_A\ \ E_{W_1}\ \ E_{W_2}\ \ E_{W_3}\ \ E_{D_1}\ \ E_{D_2}\ \ E_{D_3}],\end{aligned}$$

in which M, E_A, E_{W_k} and E_{D_k} $(k=1,2,3)$ are known constant matrices and $\nabla(t)$ is an unknown matrix satisfying

$$\nabla^{\mathrm{T}}(t)\nabla(t)\le I,$$

$G^{(k)}=(g_{ij}^{(k)})_{N\times N}$ $(k=1,2,3)$ are the out-coupling matrices representing the coupling strength and the topological structure of the networks and satisfying the diffusive coupling connections:

$$g_{ij}^{(k)}=g_{ji}^{(k)}\ge 0\ (i\ne j),\quad g_{ii}^{(k)}=-\sum_{j=1,j\ne i}^{N} g_{ij}^{(k)}\ (i,j=1,2,\ldots,N).$$

In this paper, the main contributions are given as follows.

(1) It is the first time to establish the robust exponential synchronization criteria for an array of hybrid coupled neural networks with parameter uncertainties, leakage delay, time-varying discrete and distributed delays. It is worth pointing out that the addressed system includes many neural network models as its special cases (see [8,10,22,37]) and some valuable mathematical techniques have been employed, which generalize the previous results to some extent.

(2) A novel augmented LKF is proposed to analyze the synchronization problem for the coupled neural networks. This method includes a new type of augmented matrices with multiple Kronecker product operations, and thus introduces more relaxed variables to alleviate the requirements of a positive definite matrix. Hence, it can effectively reduce the conservatism of the synchronization criteria.

(3) The proposed control method removes the traditional restriction on slowly varying delay. Using this method, even if the information about the derivative of the time-varying delay $h(t)$ is unknown, delay-derivative-independent synchronization criteria can be derived.

The organization of this paper is as follows. In Section 2, some preliminaries are given. In Section 3, synchronization criteria are derived for the CNNs without uncertainties. In Section 4, sufficient conditions are presented for the robust synchronization of system (1.1). In Section 5, some numerical examples are provided to illustrate the effectiveness of the obtained theoretical results. A brief remark is given in Section 6 to conclude this work.

2. Preliminaries

For convenience, we introduce several notations. R^n is the n-dimensional Euclidean space, $R^{m\times n}$ denotes the set of $m\times n$ real matrices, I_n represents the n-dimensional identity matrix, $A>(\ge)0$ means that A is a symmetric positive definite (semidefinite) matrix, $\lambda_{\max}(A)$ denotes the largest eigenvalue of A, $A\otimes B$ stands for the Kronecker product of matrices A and B, for $B=(B_{ij})_{(mp)\times(nq)}$, $B_{ij}\in R^{m\times n}$ $(1\le i\le p,\ 1\le j\le q)$, $A^{\otimes B}$ stands for

$$\begin{bmatrix} A\otimes B_{11} & \cdots & A\otimes B_{1q}\\ \vdots & \ddots & \vdots\\ A\otimes B_{p1} & \cdots & A\otimes B_{pq}\end{bmatrix},$$

for a vector $x=(x_1,x_2,\ldots,x_n)^{\mathrm{T}}\in R^n$, $\|x\|$ stands for the Euclidean norm, $*$ denotes the symmetric block in a symmetric matrix. Matrix dimensions, if not explicitly stated, are assumed to be compatible for algebraic operations.

Combined with the sign $\otimes$ of the Kronecker product, (1.1) can be rewritten as

$$\begin{aligned}\dot{x}(t) &= -(I_N\otimes\hat{A})x(t-\tau)+(I_N\otimes\hat{W}_1)F(x(t))+(I_N\otimes\hat{W}_2)F(x(t-h(t)))\\ &\quad +(I_N\otimes\hat{W}_3)\int_{t-\sigma(t)}^{t}F(x(s))\,ds+U(t)+(G^{(1)}\otimes\hat{D}_1)x(t)\\ &\quad +(G^{(2)}\otimes\hat{D}_2)x(t-h(t))+(G^{(3)}\otimes\hat{D}_3)\int_{t-\sigma(t)}^{t}x(s)\,ds, \qquad (2.1)\end{aligned}$$

where $x(\cdot)=(x_1^{\mathrm{T}}(\cdot),x_2^{\mathrm{T}}(\cdot),\ldots,x_N^{\mathrm{T}}(\cdot))^{\mathrm{T}}$, $F(x(\cdot))=(f^{\mathrm{T}}(x_1(\cdot)),f^{\mathrm{T}}(x_2(\cdot)),\ldots,f^{\mathrm{T}}(x_N(\cdot)))^{\mathrm{T}}$, and $U(t)=(u^{\mathrm{T}}(t),u^{\mathrm{T}}(t),\ldots,u^{\mathrm{T}}(t))^{\mathrm{T}}$.

Throughout this paper, we make the following assumptions.

(H1) The time delays satisfy

$$\tau\ge 0,\quad 0\le h_m\le h(t)\le h_M,\quad \dot{h}(t)\le h_D,\quad 0\le\sigma(t)\le\sigma,$$

where h_m, h_M, h_D and σ are known constant scalars.

(H2) For any y_1, $y_2\in R$, there exist constants δ_r^- and δ_r^+, such that the activation functions satisfy

$$\delta_r^-\le\frac{f_r(y_1)-f_r(y_2)}{y_1-y_2}\le\delta_r^+,\quad r=1,2,\ldots,n.$$

We denote

$$\bar{h}=\begin{cases} h_M & \text{if } h_D\le 1\\ h_m & \text{if } h_D>1\end{cases},\quad \Delta_1=\mathrm{diag}\left\{\delta_1^-\delta_1^+,\ldots,\delta_n^-\delta_n^+\right\},$$

$$\Delta_2=\mathrm{diag}\left\{\frac{\delta_1^-+\delta_1^+}{2},\ldots,\frac{\delta_n^-+\delta_n^+}{2}\right\}.$$

Next, we give some useful definitions and lemmas.

Definition 2.1. The CNNs (1.1) are said to be robustly globally exponentially synchronized if for all admissible uncertainties $\Delta A(t)$, $\Delta W_k(t)$, $\Delta D_k(t)$ $(k=1,2,3)$, there exist $\eta>0$ and $M_0>0$