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Research Article

Stability analysis for discrete-time switched systems with unstable subsystems by a mode-dependent average dwell time approach

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ABSTRACT

This paper mainly intends to present new stability results of a discrete-time switched system with unstable subsystems. By adopting multiple Lyapunov functions' (MLFs') method, new and less conservative stability conditions are derived in terms of a set of numerical feasible linear matrix inequalities (LMIs) with mode-dependent average dwell time (MDADT) techniques. Different from previous literatures, unstable subsystems are considered under two situations in this paper. It is shown that the discrete-time switched system can achieve exponential stability under a slow switching scheme and even in the presence of fast switching of unstable subsystems. Finally a numerical example is given to demonstrate the effectiveness of the proposed method.

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1. Introduction

In the past decades, switched systems have drawn considerable attentions and interests in control field [1–5,15–17,23]. Such systems are an important class of hybrid systems. Typically, switched systems are composed of a finite number of continuous-time or discrete-time subsystems and the corresponding switching signal orchestrating the switching between them. In real world, due to abrupt change of environment and intelligent control requirements, larger numbers of systems, e.g., automobile engine control system, flight control system, network control system, are modeled into switched systems [6–8]. So far, there are numerous sound and pioneered results related to switched systems [9–11], in particular, stability analysis and controller synthesis are the focuses of switched systems.

To seek less conservative results is a fundamental problem in stability analysis. Switched systems, without exception, possess stability analysis problem. So far, two stability issues have been addressed, i.e., the stability under arbitrary switching and the stability under constrained switching. As for the former case, a common Lyapunov function method is adopted to satisfying all subsystems [1]. On the other hand, for the latter case, it is shown that multiple Lyapunov functions' (MLFs) approach has more freedom to analyze the stability of switched systems under constrained switching [2]. In a certain sense, switched systems can be categorized into time-controlled switching, state-controlled switching or a combination of them [1]. Recently, much efforts and activities are centralized on time-controlled switching, i.e., see survey paper [9,10], and the references therein. Fortunately, it has produced many remarkable results in time-controlled switched systems' branch. A typical example is average dwell time (ADT), which is demonstrated to be a successful and effective technique to analyze switched system stability and design controllers [1,2,7,12,13,24]. At latest, the author in [6] proposed mode-dependent average dwell time (MDADT) which is shown to be a more general class of ADT.

However, the majority of above results mainly consider switched systems with all stable subsystems. For instance, these papers [18–20] considered switched systems which are all stable subsystems, whereas switched systems with unstable subsystems are inevitably encountered in real plant (e.g., sensor faults or controller failure [9,21,22] can lead to unstable subsystems). Moreover, for practical complex systems, it may be difficult and unnecessary to stabilize all subsystems. Therefore, it is meaningful to consider the case that the closed-loop switched systems might contain unstable subsystems. Paper [14] researched the switched systems with unstable subsystems by ADT techniques and obtained some beneficial results. But, in such reports, the switching scheme is mainly confined to slow switching. At very recently, the authors study the switched systems with stable and unstable subsystems in a continuous domain in paper [22], which

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presented less conservative exponentially stable conditions and the fast switching scheme. To the best of authors knowledge, there is no result about stability analysis for switched systems with unstable subsystems the discrete-time domain under slow switching and fast switching (not in the sense of arbitrary switching).

Motivated by above works, in this paper, we give our insight into discrete-time switched systems with unstable subsystems under two situations, that is **S1**: unstable subsystems are allowed to slow switching like stable subsystems, but stable subsystems have to dwell in a sufficient long time. The other situation **S2**: unstable subsystems follow the fast switching scheme. Based on MLFs' method combined with MDADT techniques, new stability results of discrete-time switched systems are obtained in terms of a set of numerical feasible linear matrix inequalities (LMIs). The remainder of this paper is organized as follows. Section 2 gives systems description and preliminaries. Stability analysis under two situations is presented in Section 3. Section 4 provides a simple numerical example to illustrate main results obtained in the last section. Finally, some ending remarks are included in Section 5.

Notations 1. The notations used are fairly standard. We use $P>0\ (\geq, <, \leq 0)$ to denote a positive define (semi-positive define, negative define, semi-negative define) matrix P. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathcal{C}^1$ denotes the space of continuously differentiable functions, and a function $\beta:[0,\infty)\to[0,\infty)$ is said to be of class $\mathcal{K}_\infty$, if it is continuous, strictly increasing, unbounded, and $\beta(0)=0$. $\|\cdot\|$ denotes the usual 2-norm. The superscript "T" stands for matrix transpose.

2. System descriptions and preliminaries

Consider the following discrete-time switched systems:

$$x(k+1)=f_{\sigma(k)}(x(k)), \quad x(k_0)=x_0. \tag{1}$$

where $x(k)\in\mathbb{R}^n$ is n-dimensional state vector. $\sigma(k)$ is a piecewise constant function of time sequence, called switching signal, which takes its value in a finite $\mathcal{I}=\{1,\dots,N\}$, N is the number of subsystems. For a switching sequence $0<k_1<k_2\cdots<k_i<k_{i+1}<\cdots$, $\sigma(k)$ is continuous from right everywhere, and let k_i^- represent the previous instant of discrete instant k_i. When $k\in[k_i,k_{i+1})$, we say that the σth(k_i) subsystem is activated. $f_{\sigma(k)}\to\{f_1,f_2,\dots,f_N\}$, each f_i is globally Lipschitz continuous. In this paper, we consider $f_{\sigma(k)}$ that can be either stable or unstable. Without losing generality, we suppose that there are $r(1\leq r\leq N)$ stable subsystems and $N-r$ unstable subsystems. For brevity, we denote $\mathcal{S}\triangleq\{1,2,\dots,r\},\mathcal{U}\triangleq\{r+1,r+2,\dots,N\}$. $\mathcal{S}$ and $\mathcal{U}$ are the sets of stable subsystems and unstable subsystems, respectively.

In this paper, we mainly consider the stability conditions of system (1) under situation **S1** and situation **S2** with MDADT switching. To end this section, we present some definitions which will be used in the following sections.

Definition 1 (*Zhang and Shi [2]*). Suppose that a switching signal $\sigma(k)$ is given. The equilibrium $x^*=0$ of system (1) is global uniformly exponential stable (GUES) under switching signal $\sigma(k)$ if for any initial conditions $x(k_0)$, there exists constant $K>0$, $0<\gamma<1$, the solution of system $x(k)$ such that

$$\|x(k)\|\leq K\gamma^{(k-k_0)}\|x(k_0)\|. \tag{2}$$

Definition 2 (*Zhao et al. [6]*). For any $k_2>k_1>0$, $p\in\mathcal{I}$, let $N_{\sigma_p}(k_2,k_1)$ denote the switching numbers that the pth subsystem is activated over the interval $[k_1,k_2]$ and $T_p(k_2,k_1)$ denote the total running time of the pth subsystem over the interval $[k_1,k_2]$, N_{0p} denote the mode-dependent chatter bounds. If there exists $\tau_{ap}>0$ such that the following holds

$$N_{\sigma_p}(k_2,k_1)\leq N_{0p}+\frac{T_p(k_2,k_1)}{\tau_{ap}}, \tag{3}$$

then we call τ_{ap} the MDADT of the pth subsystem.

Remark 1. Definition 2 defines a novel set of switching signal with MDADT. Different from ADT, $T_p(k_2,k_1)$ is not the total running time of the whole system, but the pth subsystem its own total running time which may dispersed among total running interval $[k_1,k_2]$.

Definition 3. For any $k_2>k_1>0$, let $N_\sigma^c(k_2,k_1)$ denote the switching numbers of the fast switching subsystem over the interval $[k_1,k_2]$, $T^c(k_2,k_1)$ denote the total running time of the fast switching subsystem over the interval $[k_1,k_2]$, N_0^c denote the mode-dependent chatter bounds. If there exists $\tau_a^c>0$ such that the following holds

$$N_\sigma^c(k_2,k_1)\geq N_0^c+\frac{T^c(k_2,k_1)}{\tau_a^c}, \tag{4}$$

then we call τ_a^c fast switching MDADT.

Remark 2. We give two classes MDADT for discrete-time switched systems. In existing results [1,9,10], slow switching is demonstrated to be an effective scheme for discrete-time switched systems to achieve stability. In this paper, we still adopt the slow switching concept to handle stable subsystems. For unstable subsystems, as aforementioned in the last section, two situations, **S1** and **S2**, are considered to investigate the stability of switched systems. The MDADT for switched systems under these two situations is corresponding to Definitions 2 and 3, respectively.

3. Stability analysis

In this section, based on MLFs' method, new stability results on discrete-time switched systems with MDADT will be derived under **S1** and **S2**. For derivation of main results, we first present the following two lemmas which will be conductive to obtain stability results.

Before proceeding further, we make some declarations on constructing MLFs for discrete-time switched systems. Under situation **S1**, all subsystems follow the slow switching scheme, and every subsystem owns its individual Lyapunov function $V_p(x(k)), p\in\mathcal{I}$, stability analysis under this situation will be deduced in Lemma 1. On the other hand, in the case of **S2**, all stable subsystem still follow the slow switching scheme, while unstable subsystems adopt the fast switching scheme. Under this situation, every stable subsystem has an individual Lyapunov function $V_p(x(k)), p\in\mathcal{S}$, for all unstable subsystem, a common Lyapunov function $V_c(x(k))$ is shared among them, and this situation will be explored in Lemma 2.

Lemma 1. *Consider system (1), let ς_p, $\mu_p>1$, $p\in\mathcal{I}$ be given constants. Suppose that there exist $\mathcal{C}^1$ functions $V_p(x(k)):\mathbb{R}^n\to\mathbb{R}$, and class $\mathcal{K}_\infty$ functions κ_1,κ_2, $\forall p\in\mathcal{I}$, such that*

$$\kappa_1(\|x(k)\|)\leq V_p(x(k))\leq\kappa_2(\|x(k)\|), \tag{5}$$

$$\triangle\triangle V_p(x(k))\leq\varsigma_p V_p(x(k)), \tag{6}$$

and $\forall(\sigma(k_i)=p,\sigma(k_i^-)=q)\in\mathcal{I}\times\mathcal{I}, p\neq q$, such that

$$V_p(x(k_i))\leq\mu_p V_q(x(k_i^-)), \tag{7}$$

then the system is GUES with marginal γ^* under any switching signal with the following conditions:

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