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# Multiscale reconstruction algorithm for compressed sensing

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ABSTRACT

Compressed sensing (CS) method has attracted increasing attention owing to providing a novel insight for signal and image processing technology. Acquiring high-quality reconstruction results plays a crucial role in successful applications of CS method. This paper presents a multiscale reconstruction model that simultaneously considers the inaccuracy properties on the measurement data and the measurement matrix. Based on the wavelet analysis method, the original inverse problem is decomposed into a sequence of inverse problems, which are solved successively from the largest scale to the original scale. An objective functional, that integrate the beneficial advantages of the least trimmed sum of absolute deviations (LTA) estimation and the combinational M-estimation, is proposed. An iteration scheme that incorporates the advantages of the homotopy method and the evolutionary programming (EP) algorithm is designed for solving the proposed objective functional. Numerical simulations are implemented to validate the feasibility of the proposed reconstruction method.


## 1. Introduction

In recent years, the CS method has attracted increasing attention. Essentially, CS method is based on the fact that a relatively small number of the projections of a sparse signal can contain most of its salient information, which can be recovered by an appropriate algorithm. Due to providing a new insight for signal and image processing, CS method has found potential applications in numerous fields, including the machine learning, imaging processing, wireless sensing networks, remote sensing, astronomy, signal processing and data analysis [1–10].

CS reconstruction task is essentially an inverse problem, in which reconstruction algorithms play a crucial role in real applications. Presently, the key issue on the improvement of the reconstruction quality has attracted increasing attention, and thus various algorithms have been proposed for CS reconstruction. Common CS reconstruction algorithms can be approximately divided into three categories [11]: (1) the greedy pursuits, including the orthogonal matching pursuit (OMP) [12], the stagewise OMP (StOMP) [13] and the regularized OMP (ROMP) [14], the compressive sampling matching pursuit (CoSaMP) [11], where these methods build up an approximation one step at a time by making locally optimal choices at each step; (2) the convex relaxation algorithms, including the interior-point methods [15], the gradient projection methods [16], the iterative thresholding algorithm [17], the split Bregman iteration algorithm [18] and the fast iteration shrinkage-thresholding method [19], where CS reconstruction model is cast as an optimization problem and (3) the combinatorial algorithms that acquire highly structured samples of the signal that support rapid reconstruction by group testing [11]. Owing to the complexities and particularities of CS reconstruction tasks and the urgent requirements of the complicated application problems, generally, seeking a reliable reconstruction method remains a critical problem.

Common CS reconstruction algorithms often consider the inaccurate properties on the measurement data, and thus the improvement of the reconstruction quality is restricted. It is found that in real applications the measurement matrix may be inaccurate because of physically implementing the measurement matrix in a sensor [20]. As a result, it is more reasonable to simultaneously consider the inaccurate properties on the measurement data and the measurement matrix in CS model. Additionally, CS reconstruction task is often cast as an optimization problem, and seeking a reliable optimization algorithm is crucial. Currently, local optimization algorithms have found wide applications in CS reconstruction. Unfortunately, it is hard for the local optimization algorithms to ensure a possible global optimal solution, and thus developing a global optimization algorithm is highly appropriate for CS inverse problem.

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In order to emphasize the above challenges, naturally, this paper attempts to develop a reconstruction method to improve the reconstruction quality. Main contributions are summarized as follows.

1. Differing from common CS reconstruction models, a multiscale reconstruction model that simultaneously considers the inaccurate properties on the measurement matrix and the measurement data is proposed. The original CS inverse problem is decomposed into a sequence of inverse problems by means of the wavelet analysis method, which are solved successively from the largest scale to the original scale.
2. An objective functional that integrates the beneficial advantages of the LTA estimation and the combinational M-estimation is proposed. An iterative scheme that incorporates the advantages of the homotopy method and the EP algorithm is designed for searching a possible global optimal solution.
3. Numerical simulations are implemented to validate the feasibility of the proposed reconstruction method. Especially, this paper presents a general framework for CS reconstruction, which may be useful for the solving of other related inverse problems.

The rest of this paper is organized as follows. In Section 2, the CS model is concisely introduced. In Section 3, the wavelet multiscale analysis method is introduced, and a multiscale reconstruction model that simultaneously emphasizes the inaccurate properties on the measurement matrix and the measurement data is proposed. In Section 4, the original CS reconstruction model is cast as an optimization problem, and an objective functional is proposed. Section 5 introduces the homotopy method and the EP algorithm, and an iterative scheme that integrates the advantages of the both algorithms is designed for solving the proposed objective functional. Numerical results and discussions are presented in Section 6. Finally, Section 7 presents a summary and conclusions.

## 2. CS model

In this section, the CS reconstruction model is introduced briefly, and more details on the CS theory can be found in [21–24]. If a signal $\boldsymbol{x}$ is sparse, CS model attempts to reconstruct $\boldsymbol{x}$ from just a few linear measurements of $\boldsymbol{x}$, which can be formulated by

$$\overline{\boldsymbol{y}} = \overline{\boldsymbol{\Phi}}\boldsymbol{x} \tag{1}$$

where $\overline{\boldsymbol{y}}$ is an $m \times 1$ dimensional vector indicating the linear measurements of $\boldsymbol{x}$; $\boldsymbol{x}$ indicates an $n \times 1$ dimensional vector standing for a sparse signal or image; $\overline{\boldsymbol{\Phi}}$ represents a matrix of dimension $m \times n$, which is called as the measurement matrix. Common measurement matrices include the Gaussian measurement matrix, the Binary measurement matrix and the Fourier measurement matrix, and more discussions on this issue can be found in [22].

If a signal is not sparse, however, it can be sparsely represented by the other bases, such as the wavelet bases and the Fourier bases, Eq. (1) can be rewritten as [25]

$$\overline{\boldsymbol{y}} = \overline{\boldsymbol{\Phi}}\overline{\boldsymbol{\psi}}\boldsymbol{a} \tag{2}$$

where $\boldsymbol{x} = \overline{\boldsymbol{\psi}}\boldsymbol{a}$, and $\overline{\boldsymbol{\psi}}$ is a matrix of dimension $n \times n$. It is worth emphasizing that in real applications, different basis functions will lead to different sparsity representations, which may bring different reconstruction results. More discussions on the sparsity representation methods can be traced back to [26,27].

With the consideration of the ubiquitous measurement noises in real applications, Eqs. (1) and (2) can be generalized as [25]

$$\overline{\boldsymbol{y}} = \overline{\boldsymbol{\Phi}}\boldsymbol{x} + \overline{\boldsymbol{r}} \tag{3}$$

$$\overline{\boldsymbol{y}} = \overline{\boldsymbol{\Phi}}\overline{\boldsymbol{\psi}}\boldsymbol{a} + \overline{\boldsymbol{r}} \tag{4}$$

where $\overline{\boldsymbol{r}}$ is an $m \times 1$ dimensional vector standing for the measurement noise.

In brief, the main motivation of the CS inverse problem is to estimate $\boldsymbol{x}$ from the given $\overline{\boldsymbol{\Phi}}$ and $\overline{\boldsymbol{y}}$ under the condition of satisfying sparsity assumption of $\boldsymbol{x}$, which is often cast as a L1-regularization problem [8]:

$$\min J(\boldsymbol{x}) = \frac{1}{2}\|\overline{\boldsymbol{\Phi}}\boldsymbol{x} - \overline{\boldsymbol{y}}\|^2 + \alpha \sum_{i=1}^{n} |\boldsymbol{x}_i| \tag{5}$$

where $\alpha > 0$ is the regularization parameter; $\|\cdot\|$ defines the 2-norm for a vector and $|\cdot|$ represents the absolute value operator.

## 3. Multiscale reconstruction model

### 3.1. Extension of the CS model

It is found from Eqs. (3) and (4) that the standard CS model only refers to the inaccurate property of the measurement data, and the considerations of the inaccuracy of the measurement matrix are absent. In real applications, the measurement matrix may be inaccurately derived from the physically implementing the measurement matrix in a sensor or the model approximation deviations [20], and thus it is essential to simultaneously consider such inaccuracies for the improvement of the reconstruction quality. The total least squares estimation simultaneously exploits the inaccurate properties of the measurement data and the measurement matrix, which can be described by [28]

$$(\overline{\boldsymbol{\Phi}} + \overline{\boldsymbol{E}})\boldsymbol{x} = \overline{\boldsymbol{y}} + \overline{\boldsymbol{r}} \tag{6}$$

where $\overline{\boldsymbol{E}}$ is a perturbation matrix of dimension $m \times n$. It is crucial to consider this kind of inaccuracy since it can explain the precision errors derived from physically implementing the measurement matrix in a sensor. For example, when $\overline{\boldsymbol{\Phi}}$ represents a system model, $\overline{\boldsymbol{E}}$ can model the errors derived from the assumptions on the transmission channel. Furthermore, $\overline{\boldsymbol{E}}$ can also model the deviations derived from the discretization of the domain of the analog signals and systems [20].

### 3.2. Background of wavelet transformation

The wavelet analysis is a particular representation of signals [29]. According to the multi-resolution analysis, a signal $f(t)$ is considered to belong to the scaling space $V_0$ (i.e.$f(t) \in V_0$). Implementing the multiscale decomposition yields [30,31]

$$V_0 = \psi_1 \oplus V_1 = \psi_1 \oplus \psi_2 \oplus \cdots \oplus \psi_\varpi \oplus V_\varpi \tag{7}$$

where $\psi_j$ represents the orthogonal complete place in the $j$th scale and $\varpi$ stands for a given decomposition scale.

By means of the matrix method, the discrete wavelet transformation (DWT) of a signal can be easily achieved. Given a vector $\boldsymbol{U}^0$ with the length of $N$, and its DWT can be described in the matrix notation by defining a matrix $\boldsymbol{W}_n$ of dimension $N \times N$ [32,33]:

$$\boldsymbol{W}_n = \begin{bmatrix} \boldsymbol{L}_n \\ \boldsymbol{H}_n \end{bmatrix} \tag{8}$$

where $\boldsymbol{L}_n$ and $\boldsymbol{H}_n$ represent the matrices of dimension $(N/2) \times N$, and they are called as the low-pass and the high-pass filter matrices, respectively. The product $\boldsymbol{W}_n\boldsymbol{U}^0$ decomposes the vector $\boldsymbol{U}^0$ into two components: the approximate coefficient $\boldsymbol{U}^1$ and the