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Fractional-order system identification and proportional-derivative control of a solid-core magnetic bearing

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ABSTRACT

This paper presents the application of fractional-order system identification (FOSI) and proportional-derivative (PD^{μ}) control to a solid-core magnetic bearing (MB). A practical strategy for closed-loop incommensurate FOSI along with a modified error criterion is utilized to model the MB system and a corresponding, verification experiment is carried out. Based on the identified model, integer-order (IO) PD and fractional-order (FO) PD^{μ} controllers are designed and compared with the same specifications. Besides, the relation between the two categories of controllers is discussed by their feasible control zones. Final simulation and experimental results show that the FO PD^{μ} controller can significantly improve the transient and steady-state performance of the MB system comparing with the IO PD controller.

1. Introduction

Due to the special merits such as no contact, no wear, no lubrication and adjustable dynamics, magnetic bearings (MBs) are being widely applied to a variety of industrial products, especially the high-speed rotating machinery and precise positioning systems [1–3]. Considering the inherent nonlinearity and open-loop instability of MBs, the accurate modeling and control are two essential aspects to achieve high performance requirements.

System identification is a widely used method for the modeling of MBs. Because the experimental data naturally contain the effects (e.g. eddy currents, hysteresis, etc.) neglected in the theoretical modeling, system identification can provide more accurate models. Lots of related studies can be found in [4–7]. In the literature, integer-order (IO) models were commonly adopted to characterize the dynamics of MBs. However, for the solid-core MB, several relevant studies [8–10] based on eddy current field analysis [11] show that it is a fractional-order (FO) system due to the effect of eddy currents. Therefore, fractional-order system identification (FOSI) seems a more suitable tool, though it has so far not been utilized in the modeling of MBs.

In fact, the FOSI has been utilized in the modeling of some representative FO systems such as heat transfer [12], battery charging [13], 3-D *RC* networks [14], pressurized heavy water reactor [15], and biomedical circuits and systems [16]. Moreover, various methods of FOSI are developed in time domain [17,18] and frequency domain [19–21]. In these methods, commensurate-order FO models are generally selected as the objects to be identified. Indeed, this limitation on orders can reduce the number of estimated parameters, but may lead to complex models for those incommensurate-order systems.

In this paper, therefore, a practical strategy for the FOSI using incommensurate-order models is adopted to obtain an accurate as well as simple model of a solid-core MB. Besides, a modified error criterion is utilized to find the optimal model with smaller phase errors.

For the identified FO model of the solid-core MB, an FO controller is a natural choice to enhance the system performance. In various kinds of FO controllers, FO PID controllers attract lots of attention since the extensive application of IO PID controllers in industry. For the unstable MB system, a PID-type stabilizing controller should at least involve a differential term (i.e. FO proportional-derivative (PD^{μ}) controller, FO proportional-integral-derivative ($PI^{\lambda}D^{\mu}$) controller, IO PD controller or IO PID controller) to compensate for the lagging phase. Considering the complexity of FO $PI^{\lambda}D^{\mu}$ controllers, only the FO PD^{μ} controller is designed to stabilize the MB in this paper. In [22] and [23], FO PD^{μ} controllers for a class of IO system and a class of FO system (both with zero poles) were proposed respectively based on the same design method and specifications. In the literature, the PD^{μ} controllers are compared with the IO P, PI, and PID controllers, respectively. Their results show that the PD^{μ} controller

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outperforms IO controllers. However, P- and-PI-type controllers cannot stabilize the MB system based on the above analysis. Besides, the MB model has no zero poles, and thus the systems with the PD^{μ} controller and the PID controller have different steady-state errors. As a result, it is not proper to compare the PD^{μ} controller with IO P, PI, and PID controllers for the MB system. In summary, from the perspective of the structure and function of controllers, the comparison between the PD^{μ} controller and the IO PD controller will make more sense for the MB system.

In this paper, for fair comparison, an FO PD^{μ} controller is designed and compared with an IO PD controller under the same gain crossover frequency and phase margin specifications. Besides, a class of IO PD controller simultaneously meeting gain crossover frequency and robustness to gain variations specifications is studied. By analyzing the feasible control zones of two controllers, the relation between the FO PD^{μ} controller and the IO PD controller is demonstrated. Based on this, the FO PD^{μ} and IO PD controllers are designed using two groups of specifications and compared for the identified FO MB model. The simulation and experimental results finally confirm the advantages and disadvantages of the two kinds of controllers.

This paper is organized as follows. In Section 2, a rough magnetic equivalent circuit model of the MB system is deduced, and the experimental design of the closed-loop system identification is introduced. In Sections 3 and 4, the FOSI for the MB system is carried out and the effectiveness of identified FO model is verified by simulation and experiments. Section 5 presents the design and discretization of FO PD^{μ} controllers. The relation between the FO PD^{μ} controller and the IO PD controller is also discussed. In Section 6, the simulation and experimental results are shown. Finally, conclusions are presented in Section 7.

2. Modeling and experimental design of system identification

2.1. Modeling and stabilizing controller design

A schematic of the MB studied in this paper is shown in Fig. 1. It is a current controlled homopolar MB composed of only a pair of radial MB units. The rigid rotor, stator and magnetic poles are made from solid ferromagnetic materials. The parameters and dimensions are presented in Table 1, and the corresponding physical system is shown in Fig. 2. In general, the MB system works around an equilibrium operating point (i_0, δ_0). Once the rotor deviates from the equilibrium position δ_0, the eddy current sensor measures and sends its displacement to the industrial PC. By a controller, a control signal is derived and then transformed into a control current using the power amplifier. The control current is added to the bias current i_0 to change the magnetic field, and the resulting magnetic force ultimately makes the rotor go back to its setting position. In this process, the control current is the input and the rotor displacement is the output of the MB. Obviously, the accurate modeling and control are indispensable to achieve high system performance for the MB in the whole closed loop.

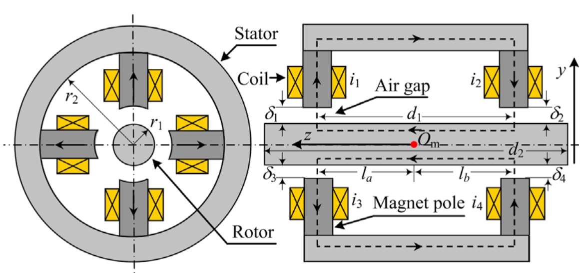

Fig. 1. Schematic of the homopolar MB and magnetic circuits.

Table 1
Parameters and dimensions of the magnetic bearing.

Symbol	Quantity	Value
N	Coil turns	344
M_r	Rotor mass	2.07 kg
A	Pole effective area	3.67×10^{-4} m^2
i_0	Bias current	0.6 A
δ_0	Nominal air gap	0.5 mm
r_1	Rotor radius	19.5 mm
r_2	Stator inner radius	57.5 mm
d_1	Distance between poles	120 mm
d_2	Rotor length	222 mm

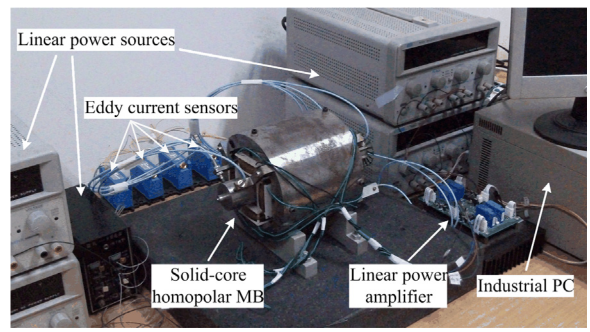

Fig. 2. Experimental platform.

Considering the serious eddy currents in the MB, system identification is utilized to obtain a precise model. However, the MB is open-loop unstable, system identification is possible only when its rotor is suspended, and thus a stabilizing controller should be first designed based on a rough model of the whole MB system model including the power amplifier, the MB and the eddy current displacement sensor in Fig. 2. Considering the high bandwidths of the amplifier (5 kHz) and the sensor (10 kHz), two constant gains can well describe their system characteristics in the work frequency range and thus only the MB is required to be modeled. In Fig. 1, the center of mass of the MB O_m is selected as the original point of displacement reference frame for modeling. The vertical direction is the y-axis, and the axial direction is the z-axis.

By magnetic equivalent circuit method (eddy currents, hysteresis and leakage are neglected), the magnetic flux ϕ_k in the k-th gap is mainly related to the k-th coil current i_k and the k-th gap length δ_k, and can be deduced as follows:

$$\phi_k = \frac{\mu_0 A N i_k}{\delta_k} \tag{1}$$

where μ_0 is the permeability of vacuum.

From the magnetic field energy in the gap and the principle of virtual displacement, the magnetic force can be calculated as

$$f_k = \frac{\phi_k^2}{2\mu_0 A} = \frac{\mu_0 A N^2}{2}\left(\frac{i_k^2}{\delta_k^2}\right) \tag{2}$$

where f_k is the magnetic force acting between the k-th magnetic pole and the rotor.

In Fig. 1, the so-called differential driving mode is used. All eight coils are divided into four couples. For the couple of coils in the y direction of the left MB unit, the upper coil current i_1 is the sum of the bias current i_0 and a control current i_{yl}, and the lower coil current i_3 is the difference $(i_0 - i_{yl})$. The gaps are $\delta_1 = \delta_0 - \delta_{yl}$, and $\delta_3 = \delta_0 + \delta_{yl}$, respectively. Linearization of (2) about the