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ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Practice Article

Identification of multivariable nonlinear systems in the presence of colored noises using iterative hierarchical least squares algorithm

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ARTICLE INFO

Article history:
Received 23 July 2011
Received in revised form
17 December 2013
Accepted 27 December 2013
This paper was recommended for publication by Dr. Q.-G. Wang

Keywords:
System identification
Nonlinear systems
Multi-input multi-output systems
Colored noises
Hammerstein model
Wiener model

ABSTRACT

This paper presents an efficient method for identification of nonlinear Multi-Input Multi-Output (MIMO) systems in the presence of colored noises. The method studies the multivariable nonlinear Hammerstein and Wiener models, in which, the nonlinear memory-less block is approximated based on arbitrary vector-based basis functions. The linear time-invariant (LTI) block is modeled by an autoregressive moving average with exogenous (ARMAX) model which can effectively describe the moving average noises as well as the autoregressive and the exogenous dynamics. According to the multivariable nature of the system, a pseudo-linear-in-the-parameter model is obtained which includes two different kinds of unknown parameters, a vector and a matrix. Therefore, the standard least squares algorithm cannot be applied directly. To overcome this problem, a Hierarchical Least Squares Iterative (HLSI) algorithm is used to simultaneously estimate the vector and the matrix of unknown parameters as well as the noises. The efficiency of the proposed identification approaches are investigated through three nonlinear MIMO case studies.

1. Introduction

In system identification, the behavior of a system is described based on its available input and output data. This ability has turned a lot of attentions and a wide variety of effective techniques have been introduced [1,2]. Most systems encountered in practice are nonlinear and linear models are not suitable to represent them. Over the years, nonlinear system identification becomes an increasingly active research area and several useful approaches have been developed [3–5]. On the other hand, most industrial systems and many control problems are inherently multi-input multi-output (MIMO). Therefore, multivariable nonlinear identification techniques are required [6–11]. Although these approaches seem quite suitable to represent many systems, they might be inadequate for the real systems where the outputs are corrupted with some noises. Most of the identification approaches address the independently distributed noises or the so-called “white” noises. However, in recent years, colored noises have taken a lot of attention [12–19]. In this aspect, some useful techniques for identification of single-input single-output (SISO) linear [12–14], SISO nonlinear [15,16], MIMO linear [17,18], and MIMO nonlinear [19] systems in the presence of colored noises are introduced.

One of the common nonlinear structures which have been widely used in many applications such as chemical processes, biological and physiological systems is the block-oriented models [20]. In particular, Hammerstein and Wiener models can be effectively used for nonlinear system identification [5,20–23]. These models consist of two cascade blocks, a linear time-invariant (LTI) subsystem and a static (memory-less) nonlinearity. In this paper, the multivariable Hammerstein and Wiener models are studied which can well represent many nonlinear systems. In the proposed approach, the nonlinear block of these multivariable models are approximated based on some known vector-based basis functions, such as polynomials, volterra series, radial basis functions or wavelets. The LTI block is modeled by an ARMAX model which can represent the moving average noises as well as the autoregressive and the exogenous dynamics. Proper combination of these two blocks leads to pseudo-linear-in-the-parameter models. Due to the multivariable nature of the system, these models contain two different kinds of unknown parameters, a vector and a matrix. Therefore, the standard least squares algorithm cannot be applied directly. Besides, the proposed models contain some terms related to the colored noises which are not known a priori. Therefore, iterative or recursive learning techniques are required to estimate them.

Recently, a new hierarchical least squares iterative (HLSI) algorithm has been introduced based on the hierarchical identification principle which can simultaneously estimate a vector and a matrix of unknown parameters [12]. Later, this iterative method was extended for identification of MIMO linear systems with

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0019-0578/$ - see front matter © 2014 ISA. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.isatra.2013.12.034

moving average noises [14]. In this paper, the HLSI algorithm is extended for nonlinear MIMO Hammerstein and Wiener systems to estimate the vector and the matrix of unknown parameters as well as the noise vectors. This paper extends the colored SISO linear models in [12–14], SISO nonlinear models in [15,16], and MIMO linear models in [17,18] to the nonlinear MIMO Hammerstein and Wiener models. In addition, since this paper considers the moving average noises, it is more general than those in [10,24]. Furthermore, the proposed method handle the MIMO systems compared to SISO ones [16,25–27].

To investigate the efficiency of the proposed identification approaches, three nonlinear multivariable systems are studied. The first one is a simulated nonlinear two-input two-output system where the outputs are correlated with some moving average noises. The other case studies are two multivariable nonlinear chemical industrial processes, a three-input three-output evaporator and a three-input six-output glass furnace processes. The data sets of these processes are taken from [28] which expresses the data is gathered from real chemical processes. The simulation results successfully demonstrate the effectiveness of the proposed multivariable nonlinear identification approaches with satisfactory performance.

The remainder of this paper is organized as follows. In Section 2, the multivariable nonlinear system identification problem is formulated as multivariable Hammerstein and Wiener models. The identification algorithms of the proposed models are presented in Section 3. Section 4 provides the identification results of the three case studies where the efficiency and applicability of the Hammerstein and Wiener models are assessed and compared. Finally, the paper concludes in Section 5.

2. Multivariable nonlinear system modeling

2.1. The Hammerstein model

Consider the multivariable Hammerstein model in Fig. 1 where $\boldsymbol{u}(t) \in \mathbb{R}^m$ and $\boldsymbol{y}(t) \in \mathbb{R}^n$ are the system input and output vectors, respectively. The zero mean white noise $\boldsymbol{v}(t) \in \mathbb{R}^n$ and the inner vector $\boldsymbol{x}(t) \in \mathbb{R}^m$ are unmeasurable [29].

Here, the linear dynamic block is described by the following autoregressive moving average with exogenous input (ARMAX) model:

$$\boldsymbol{y}(t) = \frac{\mathbf{Q}(q)}{\alpha(q)}\boldsymbol{x}(t) + \frac{D(q)}{\alpha(q)}\boldsymbol{v}(t) \tag{1}$$

where $\alpha(q)$ and $D(q)$ are polynomials in the unit time delay operator q^{-1} $[q^{-1}y(t) = y(t-1)]$ and $\mathbf{Q}(q)$ is a polynomial matrix in operator q^{-1}, which are defined as follows:

$$\alpha(q) = 1 + \alpha_1 q^{-1} + \alpha_2 q^{-2} + \ldots + \alpha_N q^{-N} \quad , \alpha_i \in \mathbb{R} \tag{2-1}$$

$$\mathbf{Q}(q) = \mathbf{Q}_1 q^{-1} + \mathbf{Q}_2 q^{-2} + \ldots + \mathbf{Q}_M q^{-M} \quad , \mathbf{Q}_i \in \mathbb{R}^{n \times m} \tag{2-2}$$

$$D(q) = 1 + d_1 q^{-1} + d_2 q^{-2} + \ldots + d_{n_d} q^{-n_d} \quad , d_i \in \mathbb{R} \tag{2-3}$$

The scalar polynomial $\alpha(q)$ is the system characteristic polynomial of order N, $\mathbf{Q}(q)$ is a polynomial matrix of order M, and the scalar polynomial $D(q)$ is of order n_d. This paper assumes that the orders N, M, and n_d are known, or can be determined by trial and error.

It is well known that nonlinear static functions can be well approximated based on basis functions expansion, such as polynomials, volterra series, radial basis functions, and wavelets [30]. Since the nonlinear block in the proposed Hammerstein model is multivariable, the bases become a vector-valued map in the corresponding multivariable model. Therefore, as it is illustrated in Fig. 2, the nonlinear multivariable block $\mathcal{N}(\cdot)$ is described by a multivariable basis function expansion as [31]:

$$\mathcal{N}(\boldsymbol{u}(t)) = \sum_{i=1}^{r} \boldsymbol{a}_i \boldsymbol{g}_i(\boldsymbol{u}(t)) \tag{3}$$

where $\boldsymbol{g}_i(\cdot): \mathbb{R}^m \to \mathbb{R}^m$, $(i = 1, \ldots, r)$ are arbitrary vactor-based known basis functions. Therefore, the unknown coefficients of these basis functions are matrices $\boldsymbol{a}_i \in \mathbb{R}^{m \times m}$, $(i = 1, \ldots, r)$.

By using (2) and (3), (1) can be rewritten as

$$\boldsymbol{y}(t) + \sum_{i=1}^{N} \alpha_i \boldsymbol{y}(t-i) = \sum_{i=1}^{M} \mathbf{Q}_i \mathcal{N}(\boldsymbol{u}(t-i)) + \sum_{i=1}^{n_d} d_i \boldsymbol{v}(t-i) + \boldsymbol{v}(t) \tag{4-1}$$

or,

$$\boldsymbol{y}(t) + \sum_{i=1}^{N} \alpha_i \boldsymbol{y}(t-i) - \sum_{i=1}^{n_d} d_i \boldsymbol{v}(t-i) = \sum_{i=1}^{M} \sum_{j=1}^{r} \mathbf{Q}_i \boldsymbol{a}_j \boldsymbol{g}_j(\boldsymbol{u}(t-i)) + \boldsymbol{v}(t) \tag{4-2}$$

This equation can be re-expressed as the following pseudo-linear-in-the-parameter model:

$$\boldsymbol{y}(t) + \boldsymbol{\psi}(t)\boldsymbol{\vartheta} = \boldsymbol{\theta}^{\mathrm{T}} \boldsymbol{\varphi}(t) + \boldsymbol{v}(t). \tag{5}$$

Definition of parameter vector $\boldsymbol{\vartheta}$, parameter matrix $\boldsymbol{\theta}$, information matrix $\boldsymbol{\psi}$, and information vector $\boldsymbol{\varphi}$ are as follows:

$$\boldsymbol{\vartheta} := [\alpha_1, \ldots, \alpha_N, d_1, \ldots, d_{n_d}]^{\mathrm{T}} \in \mathbb{R}^{N+n_d} \tag{6}$$

$$\boldsymbol{\theta}^{\mathrm{T}} := [\boldsymbol{\beta}_{11}, \ldots, \boldsymbol{\beta}_{1r}, \boldsymbol{\beta}_{21}, \ldots, \boldsymbol{\beta}_{2r}, \ldots, \boldsymbol{\beta}_{M1}, \ldots, \boldsymbol{\beta}_{Mr}] \in \mathbb{R}^{n \times Mmr} \tag{7}$$

$$\boldsymbol{\psi}(t) := [\boldsymbol{y}(t-1), \ldots, \boldsymbol{y}(t-N), -\boldsymbol{v}(t-1), \ldots, -\boldsymbol{v}(t-n_d)] \in \mathbb{R}^{n \times (N+n_d)} \tag{8}$$

$$\boldsymbol{\varphi}(t) := [\boldsymbol{g}_1^T(\boldsymbol{u}(t-1)), \ldots, \boldsymbol{g}_r^T(\boldsymbol{u}(t-1)), \ldots, \boldsymbol{g}_1^T(\boldsymbol{u}(t-M)), \ldots, \boldsymbol{g}_r^T(\boldsymbol{u}(t-M))]^{\mathrm{T}} \in \mathbb{R}^{Mmr} \tag{9}$$

where

$\boldsymbol{\beta}_{ij} = \mathbf{Q}_i \boldsymbol{a}_j \in \mathbb{R}^{n \times m}$, $i = 1, \ldots, M, j = 1, \ldots, r.$

In the above model, the objective is to identify the parameters in (6) and (7). $\boldsymbol{\beta}_{ij}$ in (7) are the multiplication of $\mathbf{Q}_i$ and $\boldsymbol{a}_j$. The parameter matrices $\mathbf{Q}_i$, $(i = 1, \ldots, M)$, and $\boldsymbol{a}_j$, $(j = 1, \ldots, r)$ cannot be separated explicitly, since any $\mathbf{Q}_i \boldsymbol{\lambda}$ and $\boldsymbol{\lambda}^{-1} \boldsymbol{a}_j$ for some nonsingular matrix $\boldsymbol{\lambda} \in \mathbb{R}^{m \times m}$ could be a solution of (5). Therefore, $\mathbf{Q}_i$ and $\boldsymbol{a}_j$ are identified up to a constant coefficient of their original

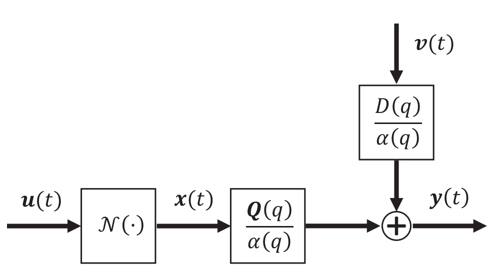

Fig. 1. Multivariable Hammerstein model.

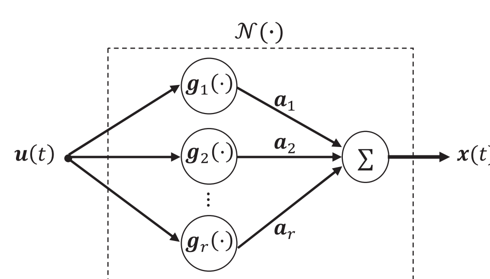

Fig. 2. Multivariable basis functions expansion.