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Highly noise-tolerant hybrid algorithm for phase retrieval from a single-shot spatial carrier fringe pattern

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ABSTRACT

A highly noise-tolerant hybrid algorithm (***NTHA***) is proposed in this study for phase retrieval from a single-shot spatial carrier fringe pattern (***SCFP***), which effectively combines the merits of spatial carrier phase shift method and two dimensional continuous wavelet transform (***2D-CWT***). ***NTHA*** firstly extracts three phase-shifted fringe patterns from the ***SCFP*** with one pixel malposition; then calculates phase gradients by subtracting the reference phase from the other two target phases, which are retrieved respectively from three phase-shifted fringe patterns by ***2D-CWT***; finally, reconstructs the phase map by a least square gradient integration method. Its typical characters include but not limited to: (1) doesn't require the spatial carrier to be constant; (2) the subtraction mitigates edge errors of ***2D-CWT***; (3) highly noise-tolerant, because not only ***2D-CWT*** is noise-insensitive, but also the noise in the fringe pattern doesn't directly take part in the phase reconstruction as in previous hybrid algorithm. Its feasibility and performances are validated extensively by simulations and contrastive experiments to temporal phase shift method, Fourier transform and 2D-***CWT*** methods.

1. Introduction

The demodulation of phase information from fringe patterns (***FPs***) is one of the most significant techniques for modern optics, such as laser interferometry, electronic speckle pattern interferometry, digital holography, and ***FP*** projection, etc. [1,2].

Temporal phase shift (*TPS*) is a most widely used technique at optical shop and laboratory, which is a typical spatial-time-domain analytical method [1,2]. It is well known to be highly accurate, high spatial resolution and computational inexpressive. However, it is vibration-sensitive, thus not suitable for vibration analysis or real time measurements.

The demodulation of a single-shot ***FP*** is an important complement to the ***TPS***, which just needs one ***FP*** to retrieve the phase map, making it considerably promising in some cases where just one ***FP*** could be captured rather than multiple ***FPs***, such as vibration analysis or real time measurement. Spatial carrier technique is often used for demodulating a single-shot ***FP***, which usually adds large tilt in the ***FP***, and then analyzes the spatial carrier fringe pattern (***SCFP***) to retrieve the phase by various algorithms, including mathematical transform method [3–14] and spatial carrier phase shift (***SCPS***) method [15–19].

One famous mathematical transform algorithm is Fourier transform (***FT***) method proposed by Takeda et al. [3,4], which belongs to spatial-frequency analytical technique. ***FT*** suffers from some problems, e.g., spectral leakage because of the insufficient carrier frequency and edge error because of Gibbs effects, etc. To overcome the limitations of ***FT***, time-frequency analysis techniques have been developed in recent years, such as the window ***FT*** (***WFT***) method [5–7] and the continuous wavelet transform (***CWT***) method [8–13]. ***WFT*** uses a short-time Fourier transform to analyze the ***FP*** locally by virtue of a window function (e.g., Gaussian function), and employs a ridge detecting technique to determine the phase information, which makes ***WFT*** more robust and has a better anti-noise performance. Qian et al. have made plentiful researches on ***WFT*** [5–7]. ***CWT*** is known as a mathematical microscopy with flexible time-frequency analysis windows. It has been widely used in signal processing, as well as fringe analysis [8–13]. With the concept of the wavelet ridge, one dimensional ***CWT*** (***1D-CWT***) technique was successfully introduced to analyze different ***FPs***. And ***2D-CWT*** is believed to be more robust (e.g., noise-insensitive), and could recovery the corrupted data in ***FPs*** [11–13], thus it has drawn more attention recently and various researches have been made to promote ***CWT*** more efficient and accurate. Ma and Wang proposed a concept of cover map to enhance the computational efficiency of ***2D-CWT*** by choosing a small number of discrete parameters instead of continuous dilation and rotation parameters [11–13], which puts forward ***CWT*** in reality considerably. The ***2D-WFT*** and ***2D-CWT*** techniques have been proven by Huang et al. to be are

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more tolerant to noise than ***1D-WFT, 1D-CWT, FT*** and ***TPS*** methods [14], because they always retrieve the phase information from the most reliable point (i.e., the ridge) of the local spectrum.

The second category is ***spatial carrier phase shift (SCPS) method*** [15,16], which extracts a set of phase-shifted ***FPs*** (***PSFPs***) from a signal ***SCFP*** with one pixel malposition, then retrieves the phase map by traditional phase shift algorithms. However, most of existing ***SCPS*** methods require a proper carrier frequency, such as $\pi/2$ rad/pixel; otherwise, residual errors will be produced. Recent improvements to ***SCPS*** could be found in Ref. [17–19].

A ***hybrid algorithm*** (***HA***) was proposed recently to retrieve the phase from a single shot ***SCFP***, which essentially combines ***SCPS*** and ***FT*** [20]. It is composed of three steps: (1) Extract three ***PSFPs*** from a single ***SCFP***; (2) The wrapped phase map of each ***PSFP*** is retrieved by ***FT***, then by a phase unwrapping and a subtraction operation, the phase shifts can be determined; (3) the phase map is retrieved by a least square phase shift algorithm. ***HA*** was found to be capable of mitigating edge error of ***FT***, and it does not require the background and modulation amplitude of the ***SCFP*** to be constant. Simultaneously, it also does not need the carrier frequency to be known in prior.

However, except for the three steps mentioned above, ***HA*** needs a dedicated filter to resist the noise in the ***SCFP*** for practical engineering, because the ***SCFP*** is directly used to retrieve the phase map in Step 3, thus the noise would be delivered into the phase map. The filter is found to be slightly prone to degrade the accuracy when the filter are not designed properly, and it is hard to design a universal filter for all kinds of ***FPs***. Therefore, this study devotes itself to enhance the robustness of ***HA***, especially the tolerance to noise in ***SCFPs***, and develops it to be a highly noise-tolerant hybrid algorithm (***NTHA***) circumventing the need of filter to the ***SCFP***. The improvements lie in:

(1) In Step 2 of previously ***HA***, it was found that the phase shifts are also the phase gradients exactly. Thus in this study, the phase gradients are directly used to reconstruct the phase map in Step 3 by an improved least square integration method [21]. Therefore, ***NTHA*** doesn't need the ***SCFP*** (with inevitable noise) to directly take part in the phase reconstruction of Step 3, thus it mitigates the influence of the noise, making ***NTHA*** more tolerant to noise.
(2) In Step 2, a time-frequency analytical method (**2D-*CWT***) is introduced to take place of the spatial-frequency method (***FT***) used in previous ***HA***, which exhibits to be more insensitive to noise, thus further enhance the robustness of ***NTHA*** to noise.

The details of ***NTHA*** are elaborated in Section 2. Its performances are investigated and validated by the simulations and contrastive experiments in Sections 3 and 4, with comprehensive comparisons to ***TPS, FT*** and **2D-*CWT***.

2. Procedures of the *NTHA*

2.1. Step 1. Extract three PSFPs

Step 1 of ***NTHA*** is generally same with previous ***HA***, it extracts three ***PSFPs*** from a single ***SCFP***. The intensity of a ***SCPF*** could be formulated as

$$I(x,y) = A(x,y) + B(x,y)\cos(\phi(x,y) + f_x x + f_y y) \tag{1}$$

where, $A(x, y)$ denotes the background intensity, $B(x, y)$ denotes the modulating amplitude, and $\phi(x, y)$ is the phase map to be determined, f_x f_y are carrier frequencies in X and Y directions, respectively. Define the phase map with tilt as

$$\Phi(x,y) = \phi(x,y) + f_x x + f_y y \tag{2}$$

Then, the local phase gradient (with tilt) of X and Y directions:

$$\begin{cases} G_x(x,y) = \partial\Phi(x,y)/\partial x = \partial\phi(x,y)/\partial x + f_x \\ G_y(x,y) = \partial\Phi(x,y)/\partial y = \partial\phi(x,y)/\partial y + f_y \end{cases} \tag{3}$$

Three ***PSFPs*** can be extracted from the single ***SCPF*** with just one pixel malposition as:

$$\begin{cases} I_1(x,y) = I(x,y) = A_1(x,y) + B_1(x,y)\cos(\Phi(x,y)) \\ I_2(x,y) = I(x+1,y) = A_2(x,y) + B_2(x,y)\cos(\Phi(x,y) + G_x(x,y)) \\ I_3(x,y) = I(x,y+1) = A_3(x,y) + B_3(x,y)\cos(\Phi(x,y) + G_y(x,y)) \end{cases} \tag{4}$$

where, the phase shifts of $I_2(x, y)$ and $I_3(x, y)$ to $I_1(x, y)$, are exactly phase gradients in X and Y directions, respectively. Due to the background and modulating amplitude are both slowly varying across the entire pupil, then we could assume

$$A = A_1 \approx A_2 \approx A_3, B = B_1 \approx B_2 \approx B_3 \tag{5}$$

Eq. (4) could be summarized as

$$I_n(x,y) = A(x,y) + B(x,y)\cos(\Phi(x,y) + \delta_n(x,y)) \tag{6}$$

where, $\delta_n(x, y)$ is the phase shift of n^{th} ***FP*** at (x, y), as Eq. (7).

$$\begin{cases} \delta_1(x,y) = 0 \\ \delta_2(x,y) = G_x(x,y) \\ \delta_3(x,y) = G_y(x,y) \end{cases} \tag{7}$$

2.2. Step 2. Determine phase gradients

Step 2 determines the phase gradients of the ***SCFP. CWT*** is introduced to retrieve the phase map in this study. ***CWT*** is a correlation process in which a large amount of similarity between the signal and a wavelet function would result in a large coefficient, and vice versa. That is, for a given scale at arbitrary position, the greater the modulus of wavelet coefficient is, the more similar the local frequency of the signal is to the vibration frequency of the wavelet function.

For ***FP*** analysis, the **2D-*CWT*** can be defined as:

$$\begin{aligned} W(\mathrm{u},s,\theta) &= \langle I, \psi_{u,s,\theta} \rangle = s^{-n}\int_{R^2} I(x)\psi*(s^{-1}r_{-\theta}(\mathrm{x}-\mathrm{u}))\mathrm{d}^2x \\ &= s^n\int_{R^2} \hat{I}(\mathbf{x})\hat{\psi}^*(sr_{-\theta}(\omega))e^{i\cdot\omega\cdot\mathrm{u}}\mathrm{d}^2\omega \end{aligned} \tag{8}$$

where, W is the wavelet transform coefficient, u is a translation parameter, s is a scale factor, θ is a rotation angle, $r_{-\theta}$ is a rotation matrix corresponding to θ, ψ is the ***2D*** wavelet function, ω is the frequency coordinate, n is the normalization parameter, the symbol "^" designates a ***FT*** operation, and the symbol "*" denotes the complex conjugate operation. One most widely used 2D Morlet wavelet is employed in this study, which is defined as:

$$\psi_M(x) = \exp(i\omega_0\cdot x)\exp\left(-\frac{|x|^2}{2\sigma^2}\right) \tag{9}$$

where, σ controls the width of wavelet function, and $\sigma\in[0.5,\ 1]$ is recommended for general ***FPs***. Then, after a series of deviations, the wavelet coefficient could be obtained, and we can determine the maximum wavelet coefficient at a region that is called wavelet ridge. Then, the phase map could be:

$$\phi(\mathrm{u}) = \tan^{-1}\frac{\mathrm{imag}(W(\mathrm{u})_{\mathrm{ridge}})}{\mathrm{real}(W(\mathrm{u})_{\mathrm{ridge}})} \tag{10}$$

where, "imag" and "real" mean the imaginary and real parts of a complex value. Then, reshape $\phi(\mathrm{u})$ to be same size with the ***FP***, $\phi(x, y)$ could be determined. After that, the phase gradients can be determined as Eq. (11).

$$\begin{cases} G_x(x,y) = \phi_2(x,y) - \phi_1(x,y) \\ G_y(x,y) = \phi_3(x,y) - \phi_1(x,y) \end{cases} \tag{11}$$