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Interpolation bias for the inverse compositional Gauss–Newton algorithm in digital image correlation

Yong Su, Qingchuan Zhang*, Xiaohai Xu, Zeren Gao, Shangquan Wu

CAS Key Laboratory of Mechanical Behavior and Design of Materials, Department of Modern Mechanics, University of Science and Technology of China, Hefei, 230027, China

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ABSTRACT

It is believed that the classic forward additive Newton–Raphson (FA-NR) algorithm and the recently introduced inverse compositional Gauss–Newton (IC-GN) algorithm give rise to roughly equal interpolation bias. Questioning the correctness of this statement, this paper presents a thorough analysis of interpolation bias for the IC-GN algorithm. A theoretical model is built to analytically characterize the dependence of interpolation bias upon speckle image, target image interpolation, and reference image gradient estimation. The interpolation biases of the FA-NR algorithm and the IC-GN algorithm can be significantly different, whose relative difference can exceed 80%. For the IC-GN algorithm, the gradient estimator can strongly affect the interpolation bias; the relative difference can reach 178%. Since the mean bias errors are insensitive to image noise, the theoretical model proposed remains valid in the presence of noise. To provide more implementation details, source codes are uploaded as a supplement.

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1. Introduction

Digital image correlation (DIC) is a non-contacting and non-interferometric full-filed optical metrology for shape, motion, and deformation measurements. This technique was originally proposed by Ranson and Peters in the early 1980s [1]. Since then, a significant and growing effort has been devoted to enhancing its measurement performance and to improving its computational efficiency [2–8]. Nowadays, DIC has evolved into a flexible, practical, and powerful technique with extensive applications in a diverse set of disciplines [9–14].

The key technique of DIC is the subpixel registration algorithm. The classic registration algorithm is the *forward additive Newton–Raphson* (FA-NR) algorithm [15]. Recently, the *inverse compositional Gauss–Newton* (IC-GN) algorithm was introduced to the field of DIC [16–18]. The radius of convergence of IC-GN algorithm is estimated to be about 3 pixels [19]. The IC-GN algorithm have been generalized to digital volume correlation (DVC) [20]. A weighted normalized gradient-based inverse compositional Gauss–Newton (WNG-IC-GN) algorithm was presented to overcome a non-uniform change of the image intensities [21]. Compared with the FA-NR algorithm, the IC-GN algorithm is more computational efficient due to the elimination of redundant computation. Thanks to this property, IC-GN algorithm have attracted extensive attentions and have come under the spotlight of DIC community [22–25].

The popularity of the IC-GN algorithm prompts a strong need to quantitatively characterize its performance. While the performance of the FA-NR algorithm has been thoroughly investigated [26–35], few papers have been devoted to analyzing the accuracy and precision of the more popular IC-GN algorithm [36–38]. Shao built a theoretical model for the IC-GN algorithm based on linear interpolation and the *sum of squared difference* (SSD) criterion for 1-dimensional registration, demonstrating that the IC-GN algorithm is more noise-robust due to the exclusion of *noise-induced bias* [36]. Pan generalized Shao's model to 2-dimensional registration with practical first-order shape function and various interpolation methods [37]. Recently, Chen compared FA-NR algorithm and IC-GN algorithm and presented an interesting finding that the noise-induced bias is not associate with the inherent flaw of FA-NR algorithm but with inappropriate implementation [38].

These literatures all utilized a spatial-domain based approach [36–38]. Although the spatial approach can precisely characterize the uncertainty due to image noise, it suffers three inherent deficiencies in featuring the *interpolation bias* [26]. Firstly, from an applied perspective, the interpolation bias formula derived by the spatial approach is unpractical [39]. As this formula involves subpixel intensities of the target image, a target image must be generated and interpolated, which is inconvenient and may introduce an extra bias in some cases [40]. Given a target image generated, however, an employment of the formula is still not preferable, because evaluating the interpolation bias straight-

* Corresponding author.
E-mail address: zhangqc@ustc.edu.cn (Q. Zhang).

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forwardly by a DIC program turns out to be a better choice. Secondly, the spatial approach is inapplicable to high-accuracy interpolation algorithms such as BSpline and O-MOMS [41], which belong to the *generalized interpolation* [42]. The spatial approach is generally based on an assumption that individual subpixel intensity can be interpolated using several nearby integer pixels. This assumption, however, is not satisfied if generalized interpolation is employed due to its infinite support [34]. Finally, interpolation bias formula derived by the spatial approach lacks clear physical meaning, failing to uncover the inherent nature of interpolation bias and provide more understanding. Aforementioned deficiencies for spatial approach are inherent. Therefore, it is essential to built a novel model for the interpolation bias of the IC-GN algorithm.

This work is devoted to building a more general, more practical, and more physical model for the interpolation bias of the IC-GN algorithm by using a frequency-domain based approach proposed in Ref. [31]. The frequency approach has been proven more effective in analyzing interpolation bias. Firstly, this frequency approach is able to estimate the interpolation bias using the power spectrum of the reference image, and thus there is no need to generate a target image. Secondly, this frequency approach employs the interpolation transfer function rather than the interpolation basis, and thus can be used for both traditional interpolation and generalized interpolation. Finally, this frequency approach provides a simple physical picture for interpolation bias, indicating that the interpolation bias is mainly determined by the integral of the product of the *interpolation bias kernel* and the image power spectrum. The frequency approach successfully resolves the inherent deficiencies of the spatial approach.

Unfortunately, the state-of-the-art frequency analysis is available for the FA-NR algorithm exclusively [31,43], and thus there is a need to generalize this approach to the IC-GN algorithm. Here, an argument against the generalization may arise: it is meaningless, because the FA-NR algorithm and the IC-GN algorithm are equivalent [16]. However, as this paper will show, the relative difference between the interpolation biases of the two algorithms can exceed 80%. The generalization from the FA-NR algorithm to the IC-GN algorithm is not straightforward. Different from the FA-NR algorithm, the IC-GN algorithm must estimate the gradients of the reference image. Consequently, the gradient estimator will influence the interpolation bias and, as this paper will show, the relative difference can reach 178%. It is therefore essential to thoroughly investigate the influence of gradient estimation on interpolation bias, which is a focus of this paper.

The contents of this work are as follows. In Section 2, the principles of FA-NR algorithm, IC-GN algorithm, and gradients estimation are briefly introduced. In Section 3, formulas for the interpolation bias of the IC-GN algorithm are deduced; the influence of gradient estimation is investigated. In Section 4, the theoretical analysis is verified by numerical experiments with 1-dimensional Gaussian speckle patterns and actual speckle images. In Section 5, the influence of image noise is explored. In Section 6, conclusions of this work are drawn.

2. Principles

2.1. FA-NR and IC-GN algorithm

Basic principles of FA-NR algorithm and IC-GN algorithm are schematically illustrated in Fig. 1. Let the reference image is $f(\mathbf{x})$, with $\mathbf{x}$ denotes the image coordinate. The target image $g(\mathbf{x}) = f(\mathbf{x} - \mathbf{u_0})$ is generated by a subpixel translation $\mathbf{u_0}$. If the *sum of squared difference* (SSD) criterion is employed, the estimated displacement $\mathbf{u}$ by DIC is

$$\mathbf{u} = \arg\min \sum_{\mathbf{x}\in S} [f(\mathbf{x}) - g(\mathbf{x}+\mathbf{u})]^2, \tag{1}$$

where S denotes the correlation subset.

Eq. (1) is generally solved by an iteration algorithm. The essential difference between the two algorithms is to which image the increment $\Delta\mathbf{u}$ is added. If the FA-NR algorithm is used, the increment $\Delta\mathbf{u}_\mathrm{F}$ is added to the target image $g(\mathbf{x})$ (see Fig. 1) and the new estimate $\mathbf{u}^{(n+1)}$ satisfies

$$\mathbf{u}^{(n+1)} = \mathbf{u}^{(n)} + \Delta\mathbf{u}_\mathrm{F}. \tag{2}$$

By contrast, if the IC-GN algorithm is used, the increment $\Delta\mathbf{u}_\mathrm{I}$ is added to reference image $f(\mathbf{x})$ (see Fig. 1). The increment is given by

$$\Delta\mathbf{u}_\mathrm{I} = -\left[\sum_{\mathbf{x}\in S} \nabla f^\mathrm{T}(\mathbf{x})\nabla f(\mathbf{x})\right]^{-1} \left\{\sum_{\mathbf{x}\in S} \nabla f^\mathrm{T}(\mathbf{x})\left[f(\mathbf{x}) - g(\mathbf{x}+\mathbf{u}^{(n)})\right]\right\}, \tag{3}$$

where $\nabla f(\mathbf{x})$ denotes the gradient of the reference image [18]. Then, the increment $\Delta\mathbf{u}_\mathrm{I}$ will compose with current estimate $\mathbf{u}^{(n)}$ to obtain a new increment $\Delta\mathbf{u}$ for the target image. In our simplified case, $\Delta\mathbf{u} = -\Delta\mathbf{u}_\mathrm{I}$. Therefore, The new estimate

$$\mathbf{u}^{(n+1)} = \mathbf{u}^{(n)} - \Delta\mathbf{u}_\mathrm{I}. \tag{4}$$

Because $\nabla f(\mathbf{x})$ is constant during the iteration, it can be calculated just once. Thanks to this property, the IC-GN algorithm is more computational efficient than the FA-NR algorithm. For more details, the readers are referred to [17,18,22].

2.2. Gradient estimation

Gradients can be estimated by a gradient operator or by interpolation.

Estimating gradients by a gradient operator is to convolve the original signal with a gradient operator d [44]. In the 1-dimensional case, the gradients of a sequence $f[n]$ are estimated by

$$f'[n] = (f \otimes d)[n] = \sum_{k=-\infty}^{\infty} f[k]d[n-k], \quad n \in \mathbb{Z}, \tag{5}$$

where $\otimes$ denotes convolution and the general form of $d[n]$ is

$$d[n] = [d_m, \ldots, d_1, 0, -d_1, \ldots, -d_m]. \tag{6}$$

Two examples are the central difference operator and the Barron operator [18]

$$\begin{aligned} f'_\mathrm{Central}[n] &= -\frac{1}{2}f[n-1] + \frac{1}{2}f[n+1], \quad n \in \mathbb{Z}; \\ f'_\mathrm{Barron}[n] &= \frac{1}{12}f[n-2] - \frac{8}{12}f[n-1] + \frac{8}{12}f[n+1] - \frac{1}{12}f[n+2], \quad n \in \mathbb{Z}. \end{aligned} \tag{7}$$

It is straightforward to extend it to the 2-dimensional case. For example, the simple Prewitt operator

$$d_x[m,n] = \frac{1}{2}\begin{bmatrix}1 & 0 & -1\end{bmatrix} \otimes \frac{1}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} = \frac{1}{6}\begin{bmatrix}1 & 0 & -1\\1 & 0 & -1\\1 & 0 & -1\end{bmatrix} \tag{8}$$

is a convolution of central difference operator along the x direction and average operator along the y direction.

Estimating gradient by interpolation is to construct a differential curve and then estimate the gradient using the gradient of that curve. For example, if *convolution-based interpolation* is employed, the recovery of a sampled sequence $f[n]$ is

$$f(x) = \sum_{k=-\infty}^{\infty} f[k]\varphi(x-k), \quad x \in \mathbb{R} \tag{9}$$

where $\varphi(x)$ is the *interpolation basis*, an approximation of the $\mathrm{sinc}(x)$ function [45]. Then, the gradient is estimated as

$$f'(x) = \sum_{k=-\infty}^{\infty} f[k]\varphi'(x-k), \quad x \in \mathbb{R} \tag{10}$$

3. Theoretical analysis of interpolation bias for the IC-GN algorithm

3.1. Interpolation bias for 1-dimensional registration

The widely used subpixel translation model is employed to evaluate the interpolation bias for the IC-GN algorithm. Firstly, the 1-dimensional registration is studied.

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