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Spirally polarized beams for polarimetry measurements of deterministic and homogeneous samples

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ABSTRACT

The use of spirally polarized beams (SPBs) in polarimetric measurements of homogeneous and deterministic samples is proposed. Since across any transverse plane such beams present all possible linearly polarized states at once, the complete Mueller matrix of deterministic samples can be recovered with a reduced number of measurements and small errors. Furthermore, SPBs present the same polarization pattern across any transverse plane during propagation, and the same happens for the field propagated after the sample, so that both the sample plane and the plane where the polarization of the field is measured can be chosen at will. Experimental results are presented for the particular case of an azimuthally polarized beam and samples consisting of rotated retardation plates and linear polarizers.

1. Introduction

Knowledge of the optical characteristics of materials is of great importance for a large number of applications, ranging from design and fabrication of photonic devices to noninvasive biomedical diagnostic techniques (see, for example, [1–3]).

Optical characterization of materials generally involves polarimetric measurements [4–8]. Using the Stokes formalism [9], any polarization state can be represented by a (four-element) Stokes vector. The effect of a sample on the polarization state of the light passing through it can be described by a 4×4 real matrix, known as Mueller matrix, which completely characterizes the polarimetric properties of the material [4–6]. In general, in order to get all the elements of the Mueller matrix, different states of polarization of the incident light have to be used, to complete a set of 16 measurements [4–6]. Such states are customary obtained by using polarizers and phase plates acting on an incident field. Some methods, based on polarization modulation, use rotating anisotropic optical elements to produce a relatively large set of incident polarization states in sequence and recover the sample characteristics by means of signal analysis [4,6,10]. Recently, two *Simon–Mukunda gadgets* [11,12] have been experimentally used to sequentially generate four different polarization states and to analyze the exiting light [13]. In *division-of-amplitude* polarimetry, to determine the state of polarization of the light exiting the sample, the output beam is split into four, and the replicas are simultaneously sent onto four analyzers [4,6,14,15]. When used to recover the optical properties of a sample, all these methods make use of a totally and uniformly polarized incident light.

Several techniques have been proposed to improve the polarization state generator, the analyzer, or both [11,13,16–27]. Some of them are faster or easier to apply and others do not require many changes in the experimental arrangement, so that potential problems, such as misalignments, are reduced. In particular, the use of suitably designed beams presenting a nonuniform distribution of the polarization state across their transverse profile has been proposed [21,22]. If states of polarization represented by four linearly independent Stokes vectors are present in the beam cross section, the Mueller matrix of a sample can be obtained from a relatively small number of measurements on imaging the beam profile onto the sample. However, in such measurements, the polarization distribution across the section of the incident light has to be carefully imaged onto the sample, because the polarization pattern changes after propagation and may also be altered by the imaging system.

Examples of nonuniformly polarized beams for which the evolution in propagation of their polarization pattern is known can be found elsewhere for completely coherent beams [28–32] as well as for partially coherent beams [30,33–36].

Recently, the use of a radially polarized input beam has been theoretically proposed for single-shot real-time polarimetry [26]. In particular, the authors propose the use of the entanglement of

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polarization and spatial degree of freedom of vector beams for the determination of sample Mueller matrix involving minimum number of measurements. A shortcut of the method is the necessity of using a large number of amplitude divisors which, as the authors recognize, could compromise its implementation.

In this paper, we propose the use of spirally polarized beams (SPBs) [37,38] for polarimetric measurements of homogeneous, deterministic and transparent samples. SPBs present a circularly symmetric field profile and linear, but non-uniform polarization across the transverse plane. In particular, the polarization state is axially symmetric and the electric field lines are logarithmic spirals. SPBs include, as limiting cases, radially and azimuthally polarized beams. Up to now, beams of these types have been shown to be useful in applications, such as microscopy, material processing, manipulation of atoms or molecules (see Ref. [39] and references therein).

Two features of SPBs are particular appealing for their application in polarimetry. First, across any transverse plane all possible linear polarization states are simultaneously present and, second, the transverse polarization pattern is invariant upon propagation [37,40]. In fact, as it will be explained in more detail later, dealing with homogeneous and deterministic [41] samples, it will be sufficient to measure the first three Stokes parameters of the beam emerging from the sample at a set of three different points across a transverse plane to recover the complete Mueller matrix of the sample. If the intensity values necessary to the evaluation of the Stokes parameters of the beam are detected by a CCD camera, as in the experiment we are going to present, the data corresponding to all the points of the transverse plane are simultaneously acquired, so that a huge number of point sets will be available at once. This can be used to reduce the experimental measurement standard deviation values so a higher accuracy of the experimental measurements is expected. Furthermore, due to the propagation invariance of the polarization pattern of the output beam, the transverse plane where to acquire the intensity profiles can be chosen at will.

The paper is structured as follows: in Section 2 the formalism is introduced, together with the numerical approach used to recover the Mueller matrix of the sample. Spirally polarized beams are recalled in Section 3, while Section 4 describes the proposed experimental technique. Upper bounds for the errors in the determination of the matrix elements and optimum conditions for obtaining them are given in Section 5. Experimental results, pertinent to the particular case of an azimuthally polarized beam and samples consisting of rotated retardation plates and linear polarizers, are reported in Section 6. Conclusions are given in Section 7.

2. Preliminaries: recovering the Mueller matrix

Partially polarized light can be represented by means of its Stokes parameters, S_i (with $i = 0, 1, 2, 3$) [9], which can be arranged to form a four-dimensional vector. The Stokes parameters, at any point in the space, can be related to a set of light intensities in the following way [6]:

$$\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} I_0 + I_{\pi/2} \\ I_0 - I_{\pi/2} \\ I_{\pi/4} - I_{-\pi/4} \\ I_{\lambda/4,\pi/4} - I_{\lambda/4,-\pi/4} \end{bmatrix} \tag{1}$$

where I_β and $I_{\lambda/4,\beta}$ represent the intensity at that point after a linear polarizer and after a quarter-wave plate followed by a linear polarizer, respectively. The subscript β refers to the angle formed by the transmission axis of the polarizer with the x axis, while $\lambda/4$ denotes the presence of the quarter-wave plate, having its fast axis at 0. Owing to the above relations, the Stokes parameters are apparently quantities obtainable from intensity measurements. Specifically S_0 represents the irradiance of the beam at a point; S_1 is the difference between the amount of linearly polarized light at 0 and the amount of linearly polarized light at $\pi/2$; S_2 is analogous to S_1 but considering linear polarization states at $\pi/4$ and $-\pi/4$; S_3 provides the difference of left and right circular polarization at such point.

When light passes through an optical system or sample, its polarization properties change. The output Stokes vector, $\mathbf{S}^{\text{out}}$, is related to the input Stokes vector, $\mathbf{S}^{\text{in}}$, through the 4×4 Mueller matrix, $\widehat{M}$, representing the polarization changes induced by the system:

$$\mathbf{S}^{\text{out}} = \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{bmatrix} \mathbf{S}^{\text{in}} = \widehat{M}\ \mathbf{S}^{\text{in}}. \tag{2}$$

In general, in order to determine all the elements of the sample's Mueller matrix (m_{ij}, $i, j = 0, 1, 2, 3$), at least 16 measurements have to be performed, involving the generation of four input polarization states, whose Stokes vectors must be linearly independent, and the projection of the output light onto four polarization states described by linearly independent Stokes vectors [4–6]. In some cases, however, for instance when samples present large linear retardance and negligible amounts of other polarization forms (such as some birefringent materials), a fewer number of measurements can be used, and the characterization of the sample turns out to be simpler. For example, in the *three-polarization probing method* [42,43] three polarization states are used for probing the sample and only the projections onto three polarization states are measured. In this way, only a 3×3 submatrix of the Mueller matrix can be determined, but the latter is sometimes sufficient to recover the complete Mueller matrix. For instance, in the case of linear and deterministic samples, i.e. samples that can be represented by a Jones matrix [5,41], the relations among the Mueller matrix elements allows for the complete Mueller matrix to be recovered from the knowledge of the above submatrix [43,44].

Let us see in more detail the recovering process of the Mueller matrix with three probing polarization states, supposing that all of them are linear. With such an assumption, for any incident polarization state the fourth Stokes parameter vanishes and Eq. (2) can be rewritten as

$$\mathbf{S}^{\text{out}} = \begin{bmatrix} m_{00}\ S_0^{\text{in}} + m_{01}\ S_1^{\text{in}} + m_{02}\ S_2^{\text{in}} \\ m_{10}\ S_0^{\text{in}} + m_{11}\ S_1^{\text{in}} + m_{12}\ S_2^{\text{in}} \\ m_{20}\ S_0^{\text{in}} + m_{21}\ S_1^{\text{in}} + m_{22}\ S_2^{\text{in}} \\ m_{30}\ S_0^{\text{in}} + m_{31}\ S_1^{\text{in}} + m_{32}\ S_2^{\text{in}} \end{bmatrix}. \tag{3}$$

If three different input linear polarization states are used, with linearly independent Stokes vectors $\mathbf{S}^{\text{in},\ell}$ (with $\ell = 1, 2, 3$), the measured intensities at the output are

$$\begin{bmatrix} S_0^{\text{out},1} \\ S_0^{\text{out},2} \\ S_0^{\text{out},3} \end{bmatrix} = \widehat{W} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \end{bmatrix}, \tag{4}$$

with the $\widehat{W}$ matrix defined as

$$\widehat{W} = \begin{bmatrix} S_0^{\text{in},1} & S_1^{\text{in},1} & S_2^{\text{in},1} \\ S_0^{\text{in},2} & S_1^{\text{in},2} & S_2^{\text{in},2} \\ S_0^{\text{in},3} & S_1^{\text{in},3} & S_2^{\text{in},3} \end{bmatrix}. \tag{5}$$

On inverting Eq. (4), the elements $m_{0\text{j}}$ ($j = 0, 1, 2$) are evaluated as

$$\begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \end{bmatrix} = \widehat{W}^{-1} \begin{bmatrix} S_0^{\text{out},1} \\ S_0^{\text{out},2} \\ S_0^{\text{out},3} \end{bmatrix}. \tag{6}$$

Note that when a uniformly polarized light is used, the input beam polarization state must be changed from one measurement to the following one. But, if a nonuniformly polarized beam is used as input beam, different polarization states can be found at different points of