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Compensation of nonlinear hardening effect in a nanoelectromechanical torsional resonator

L. Laurent [a,b,*], J.J. Yon [a,b], J.S. Moulet [a,b], P. Imperinetti [a,b], L. Duraffourg [a,b,*]

[a] Université Grenoble Alpes, F-38000 Grenoble, France
[b] CEA, LETI, Minatec Campus, F-38054 Grenoble, France

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ABSTRACT

A mechanical resonator based on torsional resonance has been fabricated in our facilities to sense infrared radiation. Actuation and detection are both electrostatic. Reaching phonon noise is highly desirable in order to get low noise uncooled infrared detectors. Therefore a high dynamic range has to be reached to increase as much as possible the overall signal to noise ratio and minimize the contribution of amplitudes noises (thermomechanical or electrical) to frequency noise. The dimensions of resonator body are similar to most clamped-clamped NEMS and our devices are thus also greatly affected by nonlinearities. We present here a nonlinear model for mechanical behavior around fundamental mode resonance taking account of both mechanical and electrostatic nonlinearities. The model correctly reproduces the nonlinear behavior observed for different resonator designs. We experimentally observe on some devices a compensation of hardening effects, allowing a linear operation of torsion angle up to 13°. The model provided in this work allows an engineering strategy in order to design high linear dynamic range for fundamental torsional mode.

1. Introduction

While Nano ElectroMechanical Sensors (NEMS) emerge in many applications [1–3], the assessment of expected performance of resonant mechanical sensors becomes more and more critical. Indeed, the small volume involved in NEMS motion makes their mechanical response extremely sensitive to nonlinearities, affecting directly the linear dynamic range (DR in dB) of operation. In the case of time references or sensors, the frequency stability needs to be as high as possible. For short integration time – such as in imaging applications – the noise, σ_y, is inversely proportional to the output amplitude DR [4]:

$$\sigma_y = \frac{1}{2Q}\frac{1}{SNR} \tag{1}$$

Where Q is the quality factor of the given eigenmode and SNR the signal-to-noise ratio in terms of amplitude (mechanical or electrical). Therefore, sources of nonlinearities must be identified and modeled to be as predictable as possible to fabricate low noise resonators. Many studies on flexural nonlinearities in mechanical resonators [5–9] can be found in the literature, but we notice a lack of fundamental studies on torsional eigenmodes. Existing works have shown than the limit in the DR, set by the critical amplitude either for stability reasons, which can be overcome by closed-loop control [10], or noise mixing in the carrier side bands, can be overcome by coupling two different vibrational modes [11] or by nonlinearity cancellation [6,7,12].

A nonlinear model is therefore of paramount importance in order to enhance the frequency stability, and thus the sensing performance, of torsional resonators. These devices were extensively studied as very sensitive magnetometers since external torque greatly influences the fundamental mode [13–15]. Mirror applications have also driven numerous studies on static [16] or dynamic [17] torsional motion.

From a mechanical point of view, torsional motion takes advantage of minimizing the beam stretching leading quickly to the nonlinear regime of the beam [5,18]. Therefore, we believe that a higher DR will occur by using torsional modes instead of flexural motion since a stiffening of the beam is usually not observed in torsional motion [8,13,14]. However, it is known that both the residual and the shear stress induce an elongation of the outer fibers of the beam. This leads to an additional stress that increases the total beam resistance to torsion [16,20]. These effects become important in thin films resonators [21]. Recent investigations have also shown that a bending of the torsional arms – induced by external

* Corresponding authors at: Université Grenoble Alpes, F-38000 Grenoble, France.
E-mail addresses: ludovic.laurent07@gmail.com (L. Laurent), laurent.duraffourg@cea.fr (L. Duraffourg).

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actuation such as electrostatic [16], thermal [22] or piezoelectric transductions [17] – increases the stretching energy in the torsional arm and, as a result, the nonlinear hardening effect in torsional stiffness.

Effectively, the DR of our stiffest devices is limited by hardening nonlinearities whereas electrostatic transduction softens the structure. Remarkably, we observe a cancellation of the first nonlinear terms of hardening sources in some devices which leads to a high linear DR of the deflection of the micro-mirror, up to 13° for the best devices. We present in this work a model describing both behaviors when most studies focus on either hardening [16,22] or softening behavior [13,19]. The model is compared with experimental characterizations on devices designed with different torsional stiffnesses – in order to test different thermal insulations to enhance long infrared radiation thermal sensing (LWIR, 8–14 μm range). In that context, the resonator size needs to be further shrunk and the model proposed here will enable the prediction of nonlinear behaviors of future devices and therefore an outlook of their DR.

2. Fabrication of torsional resonators

We initially design our resonator to fit the bolometer requirements, especially the thermal isolation from substrate. Thus, our sensor is isolated by long (8.6 μm) and narrow (250 nm) arms. Torsional beams perpendicular to the insulating arms are $L_r = 3.8$ μm long and $w_r = 250$ nm wide, thereby minimizing thermal conduction losses (Fig. 1a). A 150 nm (t_r) layer of amorphous silicon is used to stiffen the whole structure (arms and paddle). Electrodes are positioned 2 μm below the paddle to build an optical Fabry-Perot resonator enhancing the absorption of 8–14 μm incident EM waves. We also design a stiffer resonator with shorter torsion arms ($L_r = 1.5$ μm). The fabrication process is described in [23].

3. Nonlinear mechanical behavior modeling

3.1. Linear description

For clarity purposes, a cross section of our device is sketched up in Fig. 1b. x_1 accounts for the electrodes separation on the substrate. L_p, the transverse length, is not depicted here. The suspended membrane is biased with a polarization voltage V_b in order to read the capacitive signal between moving and fixed electrodes, whereas the actuation is done by biasing the other fixed electrode with an V_{AC} actuation voltage

A heterodyne detection scheme is experimentally used to overcome capacitive feedthrough (Fig. 1c) [24]. The capacitive signal is amplified through a home-made circuit, using a capacitance $C_{C2V} = 1\,pF$ as negative feedback (Fig. 1c), so that the motional signal (at frequency $\Delta\omega$) is expressed as:

$$|V_{out}(\Delta\omega)| = V_b \frac{\left(\Delta C\left(\theta\right) - \Delta C\left(-\theta\right)\right)}{2C_{FB}} \tag{2}$$

Where $\Delta C\left(\theta\right) = C_0 - C_d\left(\theta\right)$ and C_d the detection capacitance. To clarify this approach (especially the electrical torque expression), an 1f actuation is assumed in the following calculations: we suppose an actuation voltage $V_{AC}\cos(\omega t)$ and a continuous bias voltage V_b, so that the effective force driving the NEMS at resonance is proportional to $2V_{AC}V_b$. The transposition for heterodyne experimental set-up is straightforward and will be given before comparing modeling results with experimental characterizations. The paddle angle is governed by Eq. (3):

$$J\ddot{\theta} + b\dot{\theta} + \kappa_\theta \theta = T_e \tag{3}$$

$$T_e = \frac{1}{2}\frac{dC_a}{d\theta}\left(V_{AC}\cos(\omega t) - V_b\right)^2 + \frac{1}{2}\frac{dC_d}{d\theta}{V_b}^2 \tag{4}$$

Where J is the inertia moment of the paddle and b the damping coefficient. κ_θ is the equivalent torsional spring constant of both torsional beams, approximated by [20]:

$$\kappa_\theta = \frac{E}{(1+\nu)}\frac{t_r w_r^3}{L_r}\left(\frac{1}{3} - 0.21\frac{t_r}{w_r}\left(1 - \frac{1}{12}\frac{t_r^4}{w_r^4}\right)\right) \tag{5}$$

Considering a paddle length $W_p \gg x_1$, and $\sin\left(\theta\right) \sim \theta << \theta_{max}$ (small amplitudes approximation), both capacitances lead to:

$$C_a\left(\theta\right) \approx -C_0\frac{\theta_{max}}{\theta}\ln\left(1 - \frac{\theta}{\theta_{max}}\right) \tag{6}$$

$$C_d\left(\theta\right) \approx C_0\frac{\theta_{max}}{\theta}\ln\left(1 + \frac{\theta}{\theta_{max}}\right) \tag{7}$$

C_0 is the capacitance at rest ($0.18fF$) and $\theta_{\max}$ the maximum geometric angle $\theta_{max} \sim \sin\left(\theta_{max}\right) = g/\left(W_p/2\right) = 21°$. Taylor's expansions of capacitances and associated derivatives according to θ have been performed up to the 3rd order in θ. It leads to the following expression for the detection capacitance:

$$C_d\left(\theta\right) \sim C_0\left(1 - \sum_{k=1}^{3}\frac{(-1)^{k+1}}{k+1}\left(\frac{\theta}{\theta_{max}}\right)^k\right) \tag{8}$$

3.2. Nonlinear differential equation

Furthermore, using the Taylor's expansions of derivatives of $C\left(\theta\right)$, we discriminate in T_e the harmonic driving torque (proportional to $\cos(\omega t)$) from the nonlinear restoring terms (proportional to θ^k):

$$T_e = -\frac{C_0}{2}\frac{1}{2\theta_{max}}2V_{AC}V_b\cos(\omega t) + \frac{C_0}{2}\sum_{k=1}^{3}\frac{k+1}{k+2}$$
$$\left(\frac{V_{AC}^2}{2} + V_b^2 + (-1)^{k+1}V_b^2\right)\frac{\theta^k}{\theta_{max}^{k+1}} + o\left(\theta^3\right) \tag{9}$$

The 2ω driving term which appears when expanding Eq. (3) is not accounted here. Indeed, our heterodyne detection scheme does not make appear this harmonic term as the torque expression reads in this particular case:

$$\begin{aligned}
T_e &= \frac{1}{2}\frac{\partial C_a}{\partial\theta}(V_{AC}\cos((2\omega - \Delta\omega)t) - V_b\cos((\omega - \Delta\omega)t))^2 + \frac{1}{2}\frac{\partial C_d}{\partial\theta}(V_b\cos((\omega - \Delta\omega)t))^2 \\
&\quad \frac{\partial C_a}{\partial\theta}\frac{V_{AC}^2}{2} + (\frac{\partial C_a}{\partial\theta} + \frac{\partial C_d}{\partial\theta})\frac{V_b^2}{2} \\
&\quad \frac{\partial C_a}{\partial\theta}(\frac{V_{AC}^2}{2}\cos((4\omega - 2\Delta\omega)t)) \\
\Leftrightarrow T_e &= \frac{1}{2}\{ \\
&\quad -\frac{\partial C_a}{\partial\theta}V_{AC}V_b(\cos((3\omega - 2\Delta\omega)t) + \cos(\omega t)) \\
&\quad \frac{V_b^2}{2}\cos(2((\omega - \Delta\omega)t))(\frac{\partial C_a}{\partial\theta} + \frac{\partial C_d}{\partial\theta})
\end{aligned} \tag{10}$$

On the contrary, with a $f/2$ actuation and a direct detection scheme, the 2ω driving term can lead to a superharmonic resonance [8] which can be used to enhance the DR of the resonator when simultaneous resonances occur [9]. The presence of a periodic oscillation in the resonator stiffness at exactly twice the resonance frequency (with a 1f actuation and a direct detection scheme) can