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Coordination of multi-agent systems on interacting physical and communication topologies

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ABSTRACT

A new framework is given for coordination of multi-agent systems that are interconnected by a physical coupling digraph. The edges of this graph represent physical couplings between agents that are fixed due to dynamical interactions. On top of the physical graph, distributed control protocols are designed where the allowed communications between agents for control purposes are prescribed by a second fixed communication digraph. The physical and communication digraphs are generally different and the combination of these two graphs forms a cyber-physical system. The interactions between physical and communication graphs are the focus of this paper. We consider different interactions between two graphs, including the case when their pinned Laplacian commutes, the case of the communication graph with diagonalizable pinned Laplacian, and the case of two general graphs. Moreover, within each graph, the relations between the agents can be either collaborative or antagonistic. To capture this, the theory of bipartite consensus is used. Coordination protocols for different cases are designed that are distributed with respect to the communication graph, and overcome the detrimental effects of the signed physical graph. The proposed control methods are illustrated by simulation examples.

1. Introduction

The topic of consensus of multi-agent systems has received significant attentions due to its broad applications [1–4]. These works have concentrated on either the consensus regulation problem (leaderless consensus problem) [5–13] or the consensus tracking problem (leader-following consensus problem) [14–18]. The vast majority of the current research on consensus has focused on multi-agent systems with *a single interaction graph*. However, as the research has turned toward cyber-physical systems with more complex interaction channels, it has become interesting to consider multi-agent systems whose dynamics are interconnected by a physical coupling graph and whose control protocols are computed using the information flow prescribed in a second communication graph [19–21]. For example, in a team of mobile robots, deployment of the team for a search and rescue mission in a hazardous environment requires integration of a cyber wireless communication graph with a physical mobility graph [22]. Also the microgrid (a building block of the smart grid) is a physical electric power network joining distributed generators. The secondary level control of the microgrid is accomplished through a second communication graph [23–27], which should be designed separately from the physical electric power network. This combination of two graphs constitutes a cyber-physical system. To compute the distributed control protocols for such dynamically interconnected multi-agent systems, the interactions between the physical coupling graph and the communication graph should be taken into account.

Another common trait of most current formulations of the consensus problem is that the edge weights of the communication graphs between agents are assumed to be nonnegative. That is, most available consensus strategies only apply to multi-agent systems with collaborative interactions. However, in real applications, there may exist some antagonistic interactions between neighboring agents, which can be characterized by negative graph weights [28,29]. In different contexts such as social networks and biological systems, the meanings of positive and negative links may be different. For instance, in social networks, a positive (resp. negative) link can be associated to a friendly (resp. hostile) mutual relationship between pairs of individuals, parties and sport teams. In gene or protein regulatory networks, a positive/negative link corresponds to activation/inhibition interaction between genes or

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proteins. Efforts to understand the properties of non-collaborative or antagonistic interactions have led to development of the signed graph theory and the related structural balance theory [30,31], where each link is associated with a sign (positive or negative) indicating whether the interaction between agents is collaborative or antagonistic.

In contrast to the bulk of research on distributed control of networks on nonnegative graphs, only a handful of results are available for networks on signed graphs, see for example [32–35]. The reference [32] is the first work that extends the notion of consensus and its distributed protocol designs to single-integrator multi-agent systems with antagonistic interactions. Based on properties of signed graphs, Altafini showed that all agents on signed networks can converge to a common value which is the same for all agents in modulus but not in sign, which was termed as *bipartite consensus*. Following this line, the bipartite consensus problem under a weak connectivity assumption was investigated for a network of agents modeled as simple scalar integrators [33,34]. Afterwards, Valcher and Misra [35] addressed bipartite consensus for a group of single-input linear time-invariant (LTI) agents on undirected communication graphs, wherein it was proved that bipartite consensus can be achieved if and only if the state space model describing each agent is stabilizable. More recently, the work of [35] was extended to the general linear multi-agent systems [36,37] over directed signed graphs. However, we note that the bipartite consensus behavior in these existing studies occurs only in multi-agent systems with a single interaction graph.

In this paper, we investigate the coordination problem (including consensus and bipartite consensus) of linear multi-agent systems coupled by a physical interaction digraph $\mathcal{G}^p$ that is fixed *a priori*. The allowed information flow for the distributed control protocols is prescribed by a separate second communication digraph $\mathcal{G}^c$ which is also fixed. Therefore, in terms of recently developed multilayer networker theory [38,39], the closed-loop form of our model is well embedded into the framework of *multiplex networks* of two layers, one for the physical layer $\mathcal{G}^p$ and the other for the communication layer $\mathcal{G}^c$. Recently, the ideal of multiplex control strategy has been proposed to solve the consensus regulation problem [40–43]. Typical assumptions in [40–42] are that the agent is described by the first-integrator or double-integrator dynamics, or that the topology structure of multiplex control actions (e.g., the proportional action, the integral action and the derivative action) is assumed to be the same. These assumptions seem inconsistent with the well-recognized fact that the agent dynamics may be complex and different control layers have different intra-layer interconnection structure. In [43], Burbano and Bernardo explored the consensus problem in networks of generic linear systems by proposing a multiplex proportional-integral control strategy. However, the linear agent dynamics is constrained to a special LTI case with $B = I$ and the graphs of two different control layers are assumed to be nonnegative and undirected. Therefore, the existing results are not applicable to solve the coordination problem of multi-agent systems with directed and antagonistic interactions.

The objective of this paper is to address the following issues for LTI multi-agent systems with two directed interaction graphs and antagonistic interactions: (1) Under what conditions, a distributed control algorithm can be developed to guarantee the multi-agent system to reach consensus tracking or bipartite consensus tracking to the leader. (2) How to design this distributed control algorithm systematically. Aiming to answer this issue, the interactions of two graphs $\mathcal{G}^p$ and $\mathcal{G}^c$ form the focus of this paper. It is assumed throughout that $\mathcal{G}^p$ may have positive or negative weights. Part one (Section 3) of the paper considers the standard consensus setting where the communication graph $\mathcal{G}^c$ has nonnegative weights. Part two (Section 4) considers the bipartite consensus setting where $\mathcal{G}^c$ may have positive or negative weights. For each setting, three different cases of the interactions between $\mathcal{G}^p$ and $\mathcal{G}^c$ will be considered: (1) The two graphs have pinned Laplacian that commutes, (2) The communication graph has a simple pinned Laplacian, that is, the pinned Laplacian is diagonalizable, and (3) The case where $\mathcal{G}^p$ and $\mathcal{G}^c$ are both general digraphs. When compared to existing studies, our contributions can be summarized as follows.

(i) The coordination problem is solved for multi-agent systems that have multiplex structure of two layers (i.e., the physical layer $\mathcal{G}^p$ and communication layer $\mathcal{G}^c$). The interactions between $\mathcal{G}^p$ and $\mathcal{G}^c$ are studied and shown to influence the design of coupling gains required to achieve coordination.
(ii) A unified approach to the analysis of coordination for physically coupled systems is provided for the design of communication protocols on both nonnegative communication networks (using standard consensus) and signed communication networks (using bipartite consensus).
(iii) We show that nonnegative physical networks can help the system achieve coordination, while the detrimental effects of signed physical networks on coordination process can be overcome by designing appropriate coupling gains.

The rest of this paper is organized as follows. Section 2 presents some preliminaries and the model for coordination issues with both physical and communication networks. Sections 3 and 4 provide the main results for consensus and bipartite consensus, respectively. Section 5 is devoted to two illustrative examples. Finally, conclusions are summarized in Section 6.

2. Preliminaries and network model

2.1. Preliminaries

In this subsection, we introduce some notations and terminologies in graph theory that will be used in the following.

I_n (or O_n) denotes the $n \times n$ identity matrix (or zero matrix) (when clear from the context, we might drop the dimension subscripts); 1_n (or 0_n) denotes a vector in $\mathbb{R}^n$ with elements being all ones (or all zeros). Transpose of real matrices and conjugate transpose of complex matrices are denoted by the superscripts "T" and "H", respectively. $\|\cdot\|$ denotes the Euclidean norm or the corresponding induced matrix 2-norm. $\otimes$ represents the Kronecker product. diag$\{A_1, \ldots, A_N\}$ defines a block-diagonal matrix whose diagonal entries are $A_1, \ldots, A_N$. For a real symmetric matrix W, denote by $\lambda_{\max}(W)$ its largest eigenvalue and $\lambda_{\min}(W)$ its smallest eigenvalue. We say $W > 0$ (or $W < 0$) if the symmetric matrix W is positive (or negative) definite. For $\xi \in \mathbb{C}$, Re(ξ) means the real part of ξ.

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a weighted digraph (directed graph), where $\mathcal{V} = \{1, \ldots, N\}$ is the node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set. A directed edge e_{ij} in $\mathcal{E}$ is denoted by the ordered pair of nodes (j, i), meaning that node i can receive information from node j and node j is called a neighbor of node i. Its adjacency matrix $G = [g_{ij}] \in \mathbb{R}^{N\times N}$ is defined as: $g_{ij} \neq 0$ if $(j, i) \in \mathcal{E}$ and $g_{ij} = 0$ otherwise. The set of neighbors of node i is denoted by $N_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$. No self-loop is allowed, hence $a_{ii} = 0, \forall i \in \mathcal{V}$. A graph $\mathcal{G}$ with all $g_{ij} \geq 0$ is called a nonnegative graph, otherwise called a signed graph. A sequence of edges $(i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)$ is called a directed path from node i_1 to node i_k. A (nonnegative or signed) digraph is said to have a spanning tree if there exists a node, called the *root*, which has a directed path to every other node in the graph.

2.2. Network model

We consider a multi-agent system consisting of a leader and N followers, where the leader is labeled as agent 0 and the