Systems & Control Letters 108 (2017) 1–7

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Optimal multiple-sensor scheduling for general scalar Gauss–Markov systems with the terminal error

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ARTICLE INFO

Article history:
Received 7 August 2016
Received in revised form 15 March 2017
Accepted 27 July 2017

Keywords:
Kalman filtering
Multiple-sensor scheduling
Terminal estimation error covariance
State estimation

ABSTRACT

In this work, we study finite-horizon multiple-sensor scheduling for general scalar Gauss–Markov systems, extending previous results where only a class of systems are considered. The scheduling objective is to minimize the terminal estimation error covariance. At each time instant, only one sensor can transmit its measurement and each sensor has limited energy. Through building a comparison function and solving its monotone intervals, an efficient algorithm is designed to construct the optimal schedule. In addition, we also provide the result for selecting multiple sensors per time instant under an assumption. Examples are provided to illustrate the proposed results.

1. Introduction

With the development of wireless communication technology, wireless sensor networks (WSNs) have attracted a wide spectrum of applications such as health care, environment monitoring, smart grid [1–4]. In WSNs, a large number of sensor nodes deployed in the area of interest provide various information for observers. In many of these applications, however, sensor nodes are usually battery-powered and replacing old batteries generally is impossible [5], and the amount of energy for communication with a remote processing center is limited. In addition, the communication bandwidth may be limited and shared by multiple sensors. These factors extremely restrict the system performance.

In this context, optimal sensor scheduling problems for remote state estimation have received considerable attention in recent years. The main objective is to minimize cost functions related to the state estimation error. Considering that only one sensor out of a set of sensors can perform a measurement, Huber [6] proposed a information-based pruning algorithm to minimize the estimation error over multiple time steps. Vitus et al. [7] considered a similar problem, and provided an optimal and a suboptimal algorithm to prune the search tree of all possible sensor schedules. Joshi and Boyd [8] approximately solved one step sensor scheduling problem based on convex optimization theory. Further, Mo et al. [9] developed a multi-step sensor selection strategy to minimize an objective function related to the estimation error covariance matrix using a relaxed convex form. Gupta et al. [10] proposed a stochastic sensor selected strategy according to a probability distribution to minimize an upper bound on the expected steady-state performance. Shi and Chen [11] developed a branch and bound approach to address the optimal periodic multiple-sensor scheduling problem. In addition, aiming to the case of constrained sensors, the same authors proposed an approximation framework to solve the periodic scheduling problem and presented an upper bound on the approximation error to valuate the performance of the framework [12]. An optimal dynamic sensor energy schedule was derived by Ren et al. [13] to minimize the average estimation error covariance over a packet loss channel. To achieve the better estimation quality, event-based sensor data scheduling algorithms were proposed in [14–17].

These algorithms above mainly aim to minimize the average estimation error. However, there exist the applications in which we focus on the terminal estimation error covariance in practice. Examples include interceptors, standardized tests and other discrete events. Related studies on minimizing terminal estimation error covariance have emerged in past few years. Savage and La Scala [18] firstly presented a set of results in the context of minimizing a terminal cost for a particular class of scalar systems. Further for more general scalar systems, the explicit optimal scheduling policies with the terminal estimation error covariance for single-sensor and multiple-sensor cases respectively were provided in [19,20]. Shi et al. [21] constructed optimal power schedules to minimize a cost function of a weighted terminal estimation error and a weighted average estimation error over a packet-delaying network. Shi and Xie [22] constructed an optimal sensor

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http://dx.doi.org/10.1016/j.sysconle.2017.07.013
0167-6911/

power schedule to minimize the expected terminal estimation error covariance over a packet-dropping network.

In this paper, we consider finite-horizon multiple-sensor scheduling for state estimation. The objective is to minimize the terminal estimation error covariance for general scalar Gauss–Markov systems subject to the sensor energy and communication bandwidth constraints. This work presents two main contributions, extending the previous results by Jia et al. in [20], where the authors showed that when only a subset of the sensors perform the measurements per time instant, a good-sensor-late-broadcast (GSLB) rule performs optimally for a class of scalar Gauss–Markov systems.

(1) First, for the case of selecting one sensor, through building a comparison function and solving its monotone intervals (see Lemma 1), we design an efficient algorithm to construct the optimal schedule for general scalar Gauss–Markov systems.
(2) Second, for the case of selecting multiple sensors, we prove that the optimality of a GSLB rule presented by [20] also holds for the other class of Gauss–Markov systems under an additional assumption.

The rest of the paper is constructed as follows. In Section 2, the sensor scheduling problem is mathematically formulated. The optimal schedule is constructed then in Section 3. The extension to selecting multiple sensors per time instant is included in Section 4. Simulation examples are provided in Section 5. Some concluding remarks are given in the end.

Notations. The positive integer k is the time index. $\mathbb{R}_+$ is the set of non-negative real numbers. For functions f, f_1, f_2 with the appropriate domains, $f_1f_2(z) := f_1(f_2(z))$ and $f^t(z) := f(f^{t-1}(z))$ with $f^0(z) := z$.

2. Problem setup

Consider the following scalar Gauss–Markov system[1] :

$$x(k+1) = ax(k) + w(k), \qquad y_i(k) = c_i x(k) + v_i(k), \tag{1}$$

where $x(k)$ is the system state, $y_i(k)$ is the measurement taken by sensor i for $i \in \{1, 2, \ldots, M\}$, $w(k)$ and $v_i(k)$ are mutually uncorrelated zero-mean white Gaussian random noises with covariances $q > 0$ and $r_i > 0$ respectively. The initial condition $x(0)$ is zero-mean Gaussian with covariance $p_0 > 0$, and is uncorrelated with $w(k)$ and $v_i(k)$. In addition, $v_i(k)$ and $v_j(k)$ are mutually uncorrelated if $i \neq j$. Define $b_i := c_i^2/r_i$, thus b_i is the sensor information value and sensor i that has bigger b_i generally embraces more accurate measurement. Without loss of generality, assume $b_1 < b_2 < \cdots < b_M$. Assume $a, c \neq 0$.

Considering the communication bandwidth constraint, only one sensor can access the communication channel to transmit its measurement per time instant. The sensors are selected according to a schedule s within a time-horizon T denoted as

$$s := [s(1), s(2), \ldots, s(T)], \tag{2}$$

where $s(k) \in \{1, \ldots, M\}$, indicating the sensor index of the kth measurement scheduled within the time-horizon T. Let $\gamma_i(k)$ be the indicator function whose value (1 or 0) implies whether sensor i is selected to use the communication channel at time k. Thus we have

$$\sum_{i=1}^{M} \gamma_i(k) = 1, k = 1, 2, \ldots, T. \tag{3}$$

[1] Note that here $\{x(k)\}$ is a Gauss–Markov process whereas $\{y(k)\}$ is only a Gauss process. Eq. (1) is also referred to as a Gauss–Markov system in [18,19,22].

Consider each sensor has limited energy and assume the energy of all sensors only can send d of the measurements to the remote estimator. Let $J_i > 0$ be the available transmission times of sensor i, then $\sum_{i=1}^{M} J_i = d$. For notational simplicity, we assume $T = d$, and the cases $T > d$ and $T < d$ will be discussed later (see Remark 3).

For linear Gaussian systems, the Kalman filter is the best estimator of $x(k)$ in a minimum mean-square sense [23]. For a given schedule s, the state estimation error covariance $p(k)$ can be recursively calculated partly in information form

$$p_s(k+1) = \left[\frac{1}{\left(a^2 p_s(k) + q\right)} + b_{s(k+1)}\right]^{-1}. \tag{4}$$

Taking the limit as $r_i \to \infty$ for $i \in \{1, 2, \ldots, M\}$, which is equivalent to the case in which no measurement is taken, the update (4) can be rewritten as

$$p(k+1) = a^2 p(k) + q. \tag{5}$$

If $|a| < 1$, i.e., a is stable, $p(k)$ in (5) converges to a steady-state value $\bar{p}$ as $k \to \infty$ and satisfies

$$\bar{p} = a^2\bar{p} + q. \tag{6}$$

By (6), we have $\bar{p} = q/1 - a^2$.

Denote S as the set of all possible schedules. In this paper, we wish to find an optimal schedule $s \in S$ to minimize the terminal error covariance subject to the sensor energy and communication bandwidth constraints, i.e.,

Problem 1.

$$\min_{s \in S} \quad p_s(T)$$
$$\text{s.t.} \quad \sum_{i=1}^{M} \gamma_i(k) = 1, k = 1, 2, \ldots, T$$
$$\sum_{k=1}^{T} \gamma_i(k) = J_i, i = 1, 2, \ldots, M.$$

3. Optimal schedule

In [20], the authors showed that when $|a| \geq 1$, the optimal scheduling policy to Problem 1 is that good sensors should be scheduled as late as possible. However, they did not present the optimal schedule for the case $|a| < 1$. In this section, we will construct the optimal schedule to Problem 1 for general scalar Gauss–Markov systems.

First define functions $h, g_i, F_{i,j} : \mathbb{R}_+ \to \mathbb{R}_+$ as follows:

$$h(z) := a^2 z + q, \tag{7}$$
$$g_i(z) := \left(h^{-1}(z) + b_i\right)^{-1}, i = 1, 2, \ldots, M, \tag{8}$$
$$F_{i,j}(z) := g_i g_j(z) - g_j g_i(z), 1 \leq i < j \leq M. \tag{9}$$

Thus $h(z)$ and $g_i(z)$ equate to the time update and measurement update for Kalman filter, respectively. $F_{i,j}(z)$ can be regarded as a comparison function where two different sensors are scheduled with the reverse order in two adjacent time instants for scalar Gauss–Markov systems. Next, we will give two lemmas, which are essential to derive the optimal schedule.

Lemma 1. *$F_{i,j}(z)$ have the following three properties:*

(1) $F_{i,j}(0) > 0$.
(2) $|a| \geq 1$: *$F_{i,j}(z)$ strictly increases on $z \in [0, \infty)$.*
(3) $|a| < 1$: *$F_{i,j}(z)$ is a piecewise monotone function that strictly increases on $z \in [0, \bar{p})$ and strictly decreases on $z \in (\bar{p}, \infty)$.*