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Actuator fault estimation for discrete-time switched systems with finite-frequency

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ABSTRACT

This paper deals with the problem of actuator fault estimation observer design for discrete-time switched systems with finite-frequency. The frequencies of the fault and the unknown input disturbance are assumed to be in a finite range. A finite frequency fault estimation observer is proposed to estimate the actuator fault in the entire frequency domain. Based on the generalized Kalman–Yakubovic–Popov (KYP) lemma and the switched Lyapunov function method, the efficient conditions are obtained to achieve the fault estimation observer design. Finally, an example is given to illustrate the proposed technique.

1. Introduction

For the past few decades, a great amount of attention has been paid to propose efficient fault diagnosis and fault tolerant control techniques due to the safety, reliability and high performance demands of various complex dynamics systems [1,2]. Fault diagnosis techniques are generally composed of two steps: the first step is to detect if a fault has occurred, which is called fault detection [3]; then the following step is to localize the fault and to estimate its magnitude, which is called fault identification. Fault estimation is often a difficult task and the literature is not so abundant as for fault detection. In recent years, several approaches have been developed by using observer-based methods and solutions were given under linear matrix inequalities (LMIs) form [4–7].

Switched systems are a class of hybrid system defined by a collection of subsystems together with a switching signal which specifies how the system switches from one subsystem to another one [8,9]. Due to the importance of switched systems in various fields, such as mechanical systems, chemical process control, electrical power systems, and traffic control systems [10,11], fault diagnosis and fault tolerant control for switched system have been extensively investigated in the past few years. For example, fault detection filter design for continuous-time and discrete-time switched system was separately considered in [12,13]. Based on the descriptor observer method, sensor fault estimation issue was studied in [14], while the discrete-time case was developed in [15]. In [16], actuator fault estimation for continuous-time switched systems with state delay was considered by using an adaptive fault diagnosis observer approach. Recently, based on a reduced-order observer and the switched Lyapunov function method, actuator fault estimation for discrete-time switched systems was proposed in [17]. An observer-based fault-tolerant control method for a class of switched nonlinear systems with unfixed or fixed dwell-time periods was investigated in [18].

It is worth noting that the works cited previously are all considered in the full frequency domain. However, for some real practical plants, the fault and the unknown disturbance frequency ranges are known in advance. Under such cases, the techniques developed for full frequency design cause great conservatism because of the overdesign. Therefore, it is necessary and more practical to design the fault diagnosis observer for switched system in finite frequency domain. Some results with finite frequency domain have been shown in recent studies. For instance, [19] was concerned with the robust H_∞ filter design for discrete-time switched system in finite frequency domain. The issue of fault detection filter design in finite frequency specification was addressed in [20] for discrete-time switched systems. The fault detection filter design with H_- and H_∞ performance indexes in finite-frequency domain was investigated in [21] for discrete-time T–S fuzzy systems. In [22], an H_∞ filter with pole-placement constraint was proposed to achieve fault estimation for discrete-time T–S fuzzy systems.

In this work, the problem of actuator fault estimation observer design for discrete-time switched systems is considered. We consider that the switched system is affected by finite-frequency disturbance and actuator fault. Based on the generalized KYP lemma and the switched Lyapunov function method, an actuator fault

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Table 1
Three ranges of the disturbance and the fault frequencies.

Frequency	LF	MF	HF
Θ_d	$\lvert\vartheta_d\rvert \le \vartheta_{dl}$	$\vartheta_{dm1} \le \vartheta_d \le \vartheta_{dm2}$	$\lvert\vartheta_d\rvert \ge \vartheta_{dh}$
Θ_f	$\lvert\vartheta_f\rvert \le \vartheta_{fl}$	$\vartheta_{fm1} \le \vartheta_f \le \vartheta_{fm2}$	$\lvert\vartheta_f\rvert \ge \vartheta_{fh}$

estimation observer is designed such that the error system is asymptotically stable with an H_∞ performance index, and the results are given in the LMIs form.

The main contributions of this paper can be summarized as follows: first, a finite-frequency robust fault estimation observer design for discrete-time switched systems is proposed, which can give an accurate estimation of the actuator fault. Second, the fault estimation observer performances, i.e. robustness against the disturbance and the fault, are guaranteed by the design procedure. Finally, the entire frequency range is divided into three ranges, which greatly reduces the computational burden.

The remainder of the paper is organized as follows. In Section 2, the considered problem is described and some preliminaries are introduced. Section 3 presents the finite-frequency actuator fault estimation observer design for discrete-time switched systems. An illustrative example is included in Section 4 to show the effectiveness of the proposed techniques. Finally, the conclusion is given in Section 5.

2. Problem description and preliminaries

Consider the following discrete-time switched system:

$$\begin{cases} x(t+1) = A_{\sigma(t)}x(t) + B_{\sigma(t)}(u(t)+f(t)) + D_{1\sigma(t)}d(t) \\ y(t) = C_{\sigma(t)}x(t) + D_{2\sigma(t)}d(t) \end{cases} \tag{1}$$

where $x(t) \in R^{n_x}$ is the state vector, $u(t) \in R^{n_u}$ is the known input vector, $y(t) \in R^{n_y}$ is the measured output vector; $f(t) \in R^{n_f}$ and $d(t) \in R^{n_d}$ are, respectively, the actuator fault signal and the unknown input disturbance, which belong to $l_2[0,\infty)$ and whose frequency ranges are finite and known in advance. Here, we define that $\vartheta_d \in \Theta_d$ and $\vartheta_f \in \Theta_f$, respectively, represent the frequency of the disturbance and the fault. Θ_d and Θ_f are divided into low-frequency (LF), middle-frequency (MF), and high-frequency (HF) three ranges and shown in Table 1: where ϑ_{dl}, ϑ_{dm1}, ϑ_{dm2}, ϑ_{dh}, ϑ_{fl}, ϑ_{fm1}, ϑ_{fm2}, and ϑ_{fh} are all assumed to be known. The piecewise function $\sigma(t) : [0,\infty) \to \aleph = \{1, 2, \ldots, N\}$ is the switching signal, which specifies which subsystem is activated at the instant t. When $\sigma(t) = i$, it means that the ith subsystem is activated. The switching signal is assumed to be *a priori* unknown, but its instantaneous value is available in real time. A_i, B_i, C_i, D_{1i}, and D_{2i} are constant real matrices of appropriate dimensions.

In order to estimate the actuator fault, the following fault estimation observer is proposed:

$$\begin{cases} \hat{x}(t+1) = A_{\sigma(t)}\hat{x}(t) + B_{\sigma(t)}(u(t)+\hat{f}(t)) - L_{\sigma(t)}(\hat{y}(t)-y(t)) \\ \hat{y}(t) = C_{\sigma(t)}\hat{x}(t) \\ \hat{f}(t+1) = \hat{f}(t) - F_{\sigma(t)}(\hat{y}(t)-y(t)) \end{cases} \tag{2}$$

where $\hat{x}(t) \in R^{n_x}$ is the observer state, $\hat{y}(t) \in R^{n_y}$ is the observer output, $\hat{f}(t) \in R^{n_f}$ is the estimation of the actuator fault $f(t)$. The matrices $L_{\sigma(t)}$ and $F_{\sigma(t)}$ are the observer parameters to be determined later.

Define

$$e_x(t) = \hat{x}(t) - x(t), \quad e_f(t) = \hat{f}(t) - f(t), \quad \Delta f(t) = f(t+1) - f(t).$$

Then from the systems (1) and (2), one can get the following augmented system:

$$\begin{cases} \bar{e}(t+1) = \tilde{A}_{\sigma(t)}\bar{e}(t) + \tilde{D}_{\sigma(t)}d(t) + \bar{I}\Delta f(t) \\ e_f(t) = \bar{I}\bar{e}(t) \end{cases} \tag{3}$$

where

$$\tilde{A}_i = \bar{A}_i - \bar{L}_i\bar{C}_i,\ \tilde{D}_i = \bar{L}_iD_{2i} - \bar{D}_{1i},\ \bar{I} = \begin{bmatrix} 0 & I \end{bmatrix}, \bar{C}_i = \begin{bmatrix} C_i & 0 \end{bmatrix},$$
$$\bar{e}(t) = \begin{bmatrix} e_x(t) \\ e_f(t) \end{bmatrix},\ \bar{A}_i = \begin{bmatrix} A_i & B_i \\ 0 & I \end{bmatrix},\ \bar{L}_i = \begin{bmatrix} L_i \\ F_i \end{bmatrix}, \bar{D}_{1i} = \begin{bmatrix} D_{1i} \\ 0 \end{bmatrix}.$$

In this paper, we mainly focus on designing a finite-frequency robust fault estimation observer for the discrete-time switched systems, such that the fault estimation error is robust to the disturbance and the fault. Therefore, the problem of this paper can be formulated as follows:

- For $d(t) = 0$ and $\Delta f(t) = 0$, system (3) is asymptotically stable;
- For $\Delta f(t) = 0$, the H_∞ performance from $d(t)$ to $e_f(t)$ is less than a given positive scalar γ_d;
- For $d(t) = 0$, the H_∞ performance from $\Delta f(t)$ to $e_f(t)$ is less than a given positive scalar γ_f.

The following lemmas are recalled:

Lemma 1 (*Generalized KYP Lemma [19]*). *Consider the following discrete-time switched system Σ_e:*

$$\begin{cases} x(t+1) = \mathcal{A}_{\sigma(t)}(\lambda)x(t) + \mathcal{B}_{\sigma(t)}(\lambda)d(t) \\ e(t) = \mathcal{C}_{\sigma(t)}(\lambda)x(t) + \mathcal{D}_{\sigma(t)}(\lambda)d(t). \end{cases} \tag{4}$$

For a given symmetric matrix

$$\Pi = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \tag{5}$$

the following statements are equivalent:
(i) *The finite frequency inequality*

$$\rho_{max}(G^i_{de}(e^{j\vartheta_d})) < \gamma, \tag{6}$$

where $\rho_{max}(G^i_{de}(e^{j\vartheta_d}))$ is the maximum singular value of the transfer function matrix $G^i_{de}(e^{j\vartheta_d})$ for the ith subsystem from the noise input $d(t)$ to error output $e(t)$;
(ii) *There exist Hermitian matrix functions $P_i(\lambda)$, $Q_i(\lambda)$ satisfying $Q_i(\lambda) > 0$, and*

$$\begin{bmatrix} \mathcal{A}_i(\lambda) & \mathcal{B}_i(\lambda) \\ I & 0 \end{bmatrix}^T \Xi_i(\lambda) \begin{bmatrix} \mathcal{A}_i(\lambda) & \mathcal{B}_i(\lambda) \\ I & 0 \end{bmatrix} + \begin{bmatrix} \mathcal{C}_i(\lambda) & \mathcal{D}_i(\lambda) \\ 0 & I \end{bmatrix}^T \Pi \begin{bmatrix} \mathcal{C}_i(\lambda) & \mathcal{D}_i(\lambda) \\ 0 & I \end{bmatrix} < 0 \tag{7}$$

where

$$\Xi_i(\lambda) = \begin{bmatrix} -P_i(\lambda) & Q_i(\lambda) \\ Q_i(\lambda) & P_i(\lambda) - 2\cos(\vartheta_{dl})Q_i(\lambda) \end{bmatrix} \tag{8}$$

for low-frequency range $\lvert\vartheta_d\rvert \le \vartheta_{dl}$,

$$\Xi_i(\lambda) = \begin{bmatrix} -P_i(\lambda) & e^{j\vartheta_{dc}}Q_i(\lambda) \\ e^{-j\vartheta_{dc}}Q_i(\lambda) & P_i(\lambda) - 2\cos(\vartheta_{dw})Q_i(\lambda) \end{bmatrix},$$
$$\vartheta_{dc} = \frac{\vartheta_{dm1} + \vartheta_{dm2}}{2}, \quad \vartheta_{dw} = \frac{\vartheta_{dm1} - \vartheta_{dm2}}{2}, \tag{9}$$

for middle-frequency range $\vartheta_{dm1} < \vartheta_d < \vartheta_{dm2}$, and

$$\Xi_i(\lambda) = \begin{bmatrix} -P_i(\lambda) & -Q_i(\lambda) \\ -Q_i(\lambda) & P_i(\lambda) + 2\cos(\vartheta_{dh})Q_i(\lambda) \end{bmatrix} \tag{10}$$

for high-frequency range $\lvert\vartheta_d\rvert \ge \vartheta_{dh}$.

Remark 1. Because $\rho_{max}(G^i_{de}(e^{j\vartheta_d}))$ is the maximum singular value of the transfer function matrix $G^i_{de}(e^{j\vartheta_d})$ for the ith subsystem from the noise input $d(t)$ to the error output $e(t)$, one can get that (6)