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Global optimal consensus for multi-agent systems with bounded controls[☆]

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ABSTRACT

This paper studies the global optimal consensus problem for a multi-agent system with bounded controls. Each agent has its own objective function which is known only to itself. We focus on two multi-agent systems, the single integrator multi-agent system and the double-integrator multi-agent system. For each of these two multi-agent systems, we construct, for each agent, a bounded local control protocol that uses the information accessible to it through the communication topology underlying the multi-agent system and information of its own objective function. It is shown that these control protocols together achieve global optimal consensus for the multi-agent system, that is, all agents reaching consensus at a state that minimizes the sum of the objective functions of all agents as long as the communication topology is strongly connected and detailed balanced. Simulation results are given to illustrate the theoretical results.

1. Introduction

Distributed cooperative control achieves several different control objectives such as consensus, swarming, flocking and formation control. Among the many distributed cooperative control problems, consensus has attracted much attention because of its many applications such as mobile robots [1], autonomous underwater vehicles [2], unmanned air vehicles (UAVs) [3], and distributed sensor networks [4]. Consensus of a multi-agent system means that agents reach an agreement on their states by using the information of their neighbors obtained through a communication network. Many different consensus control protocols have been constructed for multi-agent systems with various communication topologies and under various real world constraints such as time-delays and actuator saturation.

As an extension of consensus, the optimal consensus problem for multi-agent systems, where the agents reach a consensus state that optimizes the sum of the objective functions of all agents, has been studied in recent years due to its applications in areas such as wireless networks [5]. Traditionally, this problem is solved in discrete-time setting [6–8]. With the development of cyber–physical systems, much attention has been paid to the continuous-time setting in order to design controllers which can be directly applied to practical systems such as robots and UVAs. In [9], the convergence analysis in the optimal consensus problem was considered for a fixed undirected graph. A nonlinear distributed coordination rule was presented in [10] to achieve optimal consensus under a switching directed communicating graph. The continuous-time system was studied with discrete-time communication in [11]. The optimal consensus problem in the presence of disturbances was studied in [12]. All these works focus on optimal consensus for agents with single-integrator dynamics. However, many practical systems possess double-integrator dynamics. It is thus crucial to design optimal consensus protocols for double-integrator systems. Recently, the optimal consensus problem for agents modeled with second-order integrator systems was considered in [13,14]. In [15], the optimal consensus problem for high-order multi-agent systems was considered.

Every actuator is subject to saturation due to its physical limitations. When actuator saturation occurs, a multi-agent system that was designed to achieve optimal consensus in the absence of actuator saturation might fail to do so. However, no results are available on the optimal consensus problem in the presence of actuator saturation. In contrast, there have been several results on global and semi-global consensus of multi-agent systems subject to actuator saturation. Global consensus in the presence of actuator saturation can be achieved when the agents are described by the single-integrator [16,17] and double-integrator systems [18]. Global consensus in the presence of actuator saturation can be achieved for general higher order but neutrally stable linear systems [18]. Semi-global consensus in the presence of actuator saturation was achieved in [19] for general higher order systems whose

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open loop poles are all in the closed left-half plane by using the low gain feedback design technique [20]. Semi-global output synchronization for heterogeneous agents subject to actuator saturation was considered in [21]. Recently, global consensus of general linear systems using bounded controls was achieved in [22].

In this paper, we establish the possibility of achieving global optimal consensus with bounded controls. Each agent in the group is endowed with an objective function of its own and known only to itself. The dynamics of each agent is represented by that of single-integrator or double-integrator, respectively. For each agent, we will construct a bounded optimal consensus protocol that uses the information of other agents obtained through the communication network and information of its own objective function. We will show that, under these control protocols, all agents reach a global consensus state that optimizes the global objective function, which is the sum of the objective functions of all agents, as long as the communication topology is strongly connected and detailed balanced. The main difficulty of the global optimal consensus problem over the global leader-following consensus problems [18,22] is that the optimal state to reach consensus at is unknown and needs to be determined by the optimal consensus algorithm.

The reminder of this paper is organized as follows. In Section 2, we recall some basic definitions and notation in graph theory and convex analysis. We then state the problem of global optimal consensus in Section 3. The solutions to this problem for two multi-agent systems, the single-integrator multi-agent system and the double-integrator multi-agent system, over a fixed communication topology, are presented in Sections 4 and 5, respectively. Simulation results are presented in Section 6. A brief conclusion is drawn in Section 7.

We will use standard notation. Let $\mathbb{R}^m$ denote the m dimensional Euclidean space. For a vector $a = [a_1 \ a_2 \ \cdots \ a_m]^{\mathrm{T}} \in \mathbb{R}^m$, $\|a\|$ denotes the Euclidean norm of a defined by $\|a\| = \sqrt{a^{\mathrm{T}}a} = \sqrt{\sum_{i=1}^{m}|a_i|^2}$ and $\|a\|_\infty$ denotes the infinity norm defined as $\|a\|_\infty = \max_i|a_i|$. For a matrix $A \in \mathbb{R}^{m\times m}$, $\|A\|_\infty = \max_i\sum_{j=1}^{m}|a_{ij}|$. Also, $\mathbf{1}_N = [1 \ 1 \ \cdots \ 1]^{\mathrm{T}} \in \mathbb{R}^N$, I_m denotes the $m \times m$ identity matrix, and $\otimes$ denotes the Kronecker product of matrices. For a symmetric matrix A, by $A > 0 (\geq 0)$ we mean that A is positive definite (positive semidefinite).

2. Preliminaries

Graphs are often used to represent the underlying communication topology among the agents in the study of multi-agent systems. A directed graph $\mathcal{G}_N$ consists of a finite, nonempty set of nodes $\mathcal{V} = \{\nu_1, \nu_2, \ldots, \nu_N\}$, each of them representing an agent, and a set of ordered pairs of nodes $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$, representing edges of the graph. An edge (ν_i, ν_j) in a directed graph indicates that agent j has access of the information from agent i. A directed path in a directed graph is a sequence of edges of the form $(\nu_{i_1}, \nu_{i_2}), (\nu_{i_2}, \nu_{i_3}), \ldots$. A directed graph is strongly connected if there exists a directed path between any pair of distinct nodes. Let $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ be the adjacency matrix associated with $\mathcal{G}_N$, where $a_{ij} > 0$ if $(\nu_j, \nu_i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Here we assume that $a_{ii} = 0$ for all $i = 1, 2, \ldots, N$. Let $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ be the Laplacian matrix associated with A, where $l_{ii} = \sum_{j=1}^{N}a_{ij}$ and $l_{ij} = -a_{ij}$ when $i \neq j$. A graph is said to be detailed balanced if there exist some real numbers $\omega_i > 0, i = 1, 2, \ldots, N$, such that the coupling weights of the graph satisfy $\omega_i a_{ij} = \omega_j a_{ji}$ for all $i, j = 1, 2, \ldots, N$ [23].

The communication topology we consider in this paper is described by a directed graph that satisfies the following assumption.

Assumption 1. The directed graph $\mathcal{G}_N$ is strongly connected and detailed balanced.

Define $M = \mathrm{diag}\{\omega\}L = [m_{ij}] \in \mathbb{R}^{N\times N}$, where $\mathrm{diag}\{\omega\} = \mathrm{diag}\{\omega_1, \omega_2, \ldots, \omega_N\}$, with $\omega_i > 0, i = 1, 2, \ldots, N$. By the definition of the detailed balanced graph, we have $\mathrm{diag}\{\omega\}L = L^{\mathrm{T}}\mathrm{diag}\{\omega\}$, namely $M = M^{\mathrm{T}}$. Since $L\mathbf{1}_N = 0$, $M\mathbf{1}_N = \mathrm{diag}\{\omega\}L\mathbf{1}_N = 0$. Thus, M is a valid symmetric Laplacian matrix, and we have the following lemma.

Lemma 1 ([24]). *Under Assumption 1, M is positive semidefinite and all the eigenvalues of M are nonnegative and real. Moreover, 0 is a single eigenvalue of M.*

We next review some basic knowledge of convex analysis. A function $f : \mathbb{R}^m \to \mathbb{R}$ is convex if, for any $x, y \in \mathbb{R}^m$,

$$f(\kappa x + (1-\kappa)y) \leq \kappa f(x) + (1-\kappa)f(y), \ \ \kappa \in [0, 1]. \tag{1}$$

A function is strictly convex if strict inequality holds in (1) whenever $x \neq y$ and $0 < \kappa < 1$. Strict convexity of a function can be verified by the following criteria:

(a) *First order condition:* Assume that f is differentiable. Then, f is strictly convex if and only if $(y-x)^{\mathrm{T}}(\nabla f(y) - \nabla f(x)) > 0$ for all $x, y \in \mathbb{R}^m$, $x \neq y$.
(b) *Second order condition:* Assume that f is twice differentiable, i.e., $\nabla^2 f$ exists. If $\nabla^2 f(x) > 0$ for all $x \in \mathbb{R}^m$, then f is strictly convex.

3. Problem statement

Consider a group of N agents, each described by an nth order integrator system as,

$$x_i^{(n)} = u_i, \quad i = 1, 2, \ldots, N, \tag{2}$$

where $x_i \in \mathbb{R}^m$ is the states, $x_i^{(n)} \in \mathbb{R}^m$ represents the nth derivative of x_i, $u_i \in \mathbb{R}^m$ is the bounded control input of agent i, $\|u_i\|_\infty \leq u_{\max}$ for some positive scalar $u_{\max}$.

Each agent has its own objective function $f_i(x_i) : \mathbb{R}^m \to \mathbb{R}$ to minimize. We make the following assumption on these objective functions of the agents.

Assumption 2. The objective functions $f_i : \mathbb{R}^m \to \mathbb{R}, i = 1, 2, \ldots, N$, are twice differentiable and $\nabla^2 f_i(x) > 0$ for all $x \in \mathbb{R}^m$.

The problem we are interested in is the global optimal consensus problem. Consider the multi-agent system whose agents are described by the dynamics (2). For each agent i, construct a bounded optimal consensus protocol u_i, $\|u_i\|_\infty \leq u_{\max}$, that uses the information of other agents obtained through the communication network and information of its own objective function, under which, the multi-agent system achieves consensus at a state x^* which minimizes the objective function $f(x) = \sum_{i=1}^{N}f_i(x)$, where the convex function $f_i(x) : \mathbb{R}^m \to \mathbb{R}$ is known only to agent i, that is, x^* is the solution to the following optimization problem,

$$\min_{x\in\mathbb{R}^m} f(x),$$

with

(a) $\lim_{t\to\infty}x_i(t) = x^*, i = 1, 2 \cdots, N$.
(b) $\lim_{t\to\infty}x_i^{(s)}(t) = 0, s = 1, 2, \ldots, n-1, i = 1, 2 \cdots, N$.

Since $f(x)$ is strictly convex, the optimal consensus state x^* is reached if the following optimality condition is satisfied,

$$\nabla f(x^*) = \sum_{i=1}^{N} \nabla f_i(x^*) = 0. \tag{3}$$

In this paper, we will solve the global optimal consensus problem for multi-agent systems (2) with $n = 1$ and $n = 2$, which are respectively the single integrator multi-agent system and the double integrator multi-agent system.