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State feedback stabilization of a class of uncertain nonlinear systems on non-uniform time domains

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ABSTRACT

This paper deals with the stabilization problem for a class of uncertain nonlinear systems on non-uniform time domains. Some sufficient conditions are derived to design a state feedback controller for a class of uncertain nonlinear time-varying systems under vanishing perturbations. Using some Gronwall's integral inequalities, the uniform exponential stability of the closed-loop systems on arbitrary time scales, is guaranteed. Then, based on the Lyapunov theory, new sufficient conditions are proposed to derive the controller which ensures the practical stability of the closed-loop time-invariant nonlinear uncertain system under non-vanishing perturbations. Some examples illustrate these results.

1. Introduction

The theory of dynamic equations on an arbitrary time scale was introduced in [1]. This theory was found promising because it unifies the theories of continuous-time and discrete-time systems [2,3]. As expected, once a result has been established for dynamic equations on an arbitrary time scale, this result holds for standard continuous differential equations (i.e. $\mathbb{R}$) and standard difference equations (i.e. $h\mathbb{Z}$, h is a real number). Besides these two cases, there are many interesting time scales with non-uniform step sizes (for instance, $\mathbb{T} = \{t_n\}_{n\in\mathbb{N}}$ of so-called harmonic numbers with $t_n = \sum_{k=1}^{n} \frac{1}{k}$, the union of disjoint intervals with variable length and fixed gap, the Cantor set).

This paper deals with the stabilization problem for a class of nonlinear systems on arbitrary time scales. This research has potential applications in such areas as dynamic programming [4], neural network [5], economic modeling [6] and quantum calculus [7] to name a few. Another interesting example is distributed control over network. Indeed, the time domain may be neither continuous nor uniformly discrete due to possible intermittent information transmissions for instance (for more details, one can refer to [8]).

Qualitative properties of linear systems on time scales have been studied in [9]. In [10–12], the properties of exponential stability of linear systems on time scale were discussed. A spectral characterization of exponential stability on time scale was given in [10] using the usual exponential function. In [11], sufficient conditions were derived to ensure uniform exponential stability using the concept of time scale generalized exponential function. In [12], uniform exponential stability for positive linear time-invariant systems was studied using the stability radius. Extensions to linear systems with structured perturbations and quasilinear systems have been investigated in [13–15]. Recently, the stability of switched linear systems has been studied using a common quadratic Lyapunov function [16] or a spectral characterization [17]. Stability of nonlinear systems on time scales was studied in [18,19]. Using Lyapunov functions, some conditions have been derived to guarantee the uniform and uniform asymptotic stability [18] and exponential stability [19] for dynamic equations on time scales.

Though there exist many results on dynamic equations evolving on non-uniform time domains, control theory on time scales

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is not much developed. Mainly linear systems were studied [20–22]. In [20], the controllability concept for linear systems was generalized on an arbitrary time scale. A region of exponential stabilization has been derived in [21] for time scale linear implicit systems. In [22], the exponential stabilization of linear systems was solved using some linear matrix inequalities. Realization theory was presented in [23] for time-varying linear systems and [24] for time-invariant linear systems. Nevertheless, there are not many results dealing on nonlinear systems defined on time scales. Realization theory for nonlinear systems has been studied in [25]. Necessary and sufficient conditions for the local static state feedback linearizability of nonlinear systems, defined on an homogeneous time scale, were given in [26]. In [27], it was shown that the uniform exponential stabilization of the linear approximation of a nonlinear system implies uniform exponential stabilization of the nonlinear system. Within, the local stabilization problem for nonlinear systems with vanishing perturbations was studied. Using an appropriate Lyapunov function, sufficient conditions were derived in [28], to design a controller which guarantees the uniform exponential stabilization of a class of nonlinear systems. Nevertheless, the derived conditions cannot be easily verified in practice. A linear state feedback controller, based on matrix inequalities, was proposed in [29] for a class of uncertain linear systems where the uncertainty satisfies a linear growth condition. However, for nonhomogeneous time scales, the controller gain cannot be easily computed.

The objective of this paper is to solve the stabilization problem for a class of nonlinear systems, with bounded uncertainty, on arbitrary time scales. First, sufficient conditions are derived to design a linear state feedback controller which guarantees the uniform exponential stability of the closed-loop system. The stability proof is based on some Gronwall inequalities and a Lipschitz kind condition for the perturbation. Integral inequalities are a widely used powerful tool for developing different stability properties for nonlinear systems, as shown in [11,14,29–31]. Here, we want to extend such results in order to study the uniform exponential stabilization of the considered class of nonlinear systems. Then, using some integral inequalities, the exponential stabilization problem for a class of nonlinear time-varying systems under integrable condition for the perturbation is studied. These results are based on some methods of estimation of general solution of the systems and imply that the perturbation is vanishing. The last part of the paper is devoted to the practical stabilization of a class of nonlinear time-invariant systems with bounded non-vanishing perturbations, inspired by the main result in [28]. In the latter, the problem of uniform exponential stabilization is solved for the same class of systems under bounded vanishing disturbances. Sufficient conditions are derived to design an appropriate linear state feedback controller using the Lyapunov theory. Through the paper, some simulations illustrate the proposed results.

2. Preliminaries on time scale theory

Let us first recall some basics on the theory of time scales. Generalities with applications and advances in dynamic equations on time scales are given in [2,3]. A time scale $\mathbb{T}$ is any closed subset of $\mathbb{R}$ with order and topological structure in a canonical way. Throughout this paper, the following notations will be used. Let $a \in \mathbb{T}$. We define the set $\mathbb{T}_a^+ := \{t \in \mathbb{T} : \ t \geq a\}$. Since a time scale may or not be connected, the concept of jump operator is useful to define the generalized Hilger derivative f^{Δ} of a function f defined on an arbitrary time scale $\mathbb{T}$.

Definition 1. For $t \in \mathbb{T}$, we define

- the forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$ by $\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}$,
- the backward jump operator $\rho : \mathbb{T} \to \mathbb{T}$ by $\rho(t) := \sup\{s \in \mathbb{T} : s < t\}$,
- the graininess function $\mu : \mathbb{T} \to \mathbb{R}_+$ by $\mu(t) := \sigma(t) - t$.

The jump operators σ and ρ allow the classification of points in $\mathbb{T}$ in the following way.

Definition 2. An element $t \in \mathbb{T}$ is said to be right-dense if $\sigma(t) = t$, right-scattered if $\sigma(t) > t$, left-dense if $\rho(t) = t$, left-scattered if $\rho(t) < t$, dense if $t = \sigma(t) = \rho(t)$ and isolated if $\sigma(t) > t > \rho(t)$.

If $\mathbb{T}$ has a left-scattered maximum, this element is called a degenerate point. Denote by $\mathbb{T}^{\kappa}$ the set of all non degenerate points of $\mathbb{T}$. Note that when $\sup(\mathbb{T}) = +\infty$, then $\mathbb{T}^{\kappa} = \mathbb{T}$. Given a function $f : \mathbb{T} \to \mathbb{R}$, we define the so-called Hilger derivative of f at a point $t \in \mathbb{T}^{\kappa}$ as follows.

Definition 3. Assume that f is a function and let $t \in \mathbb{T}^{\kappa}$. Then, we define the Hilger derivative $f^{\Delta}(t)$ at t to be the number (provided it exists) with the property that given any $\varepsilon > 0$, there is a neighborhood U of t such that

$$|[f(\sigma(t)) - f(s)] - f^{\Delta}(t)[\sigma(t) - s]| \leq \varepsilon|\sigma(t) - s| \quad \text{for all } s \in U.$$

Alternatively, one can define

$$f^{\Delta}(t) := \lim_{s \to t, s \neq \sigma(t)} \frac{f(\sigma(t)) - f(s)}{\sigma(t) - s}.$$

Clearly, the generalized derivative on time scales becomes the usual derivative when $\mathbb{T} = \mathbb{R}$, i.e. $f^{\Delta}(t) = \dot{f}(t)$. Furthermore, if $\mathbb{T} = \mathbb{Z}$, then $f^{\Delta}(t)$ reduces to the usual forward difference operator $f^{\Delta}(t) = \Delta f(t)$. Hence, the time scale theory unifies both differential and difference equation theories.

Definition 4. A function $F : \mathbb{T} \to \mathbb{R}$ is called antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^{\Delta}(t) = f(t)$ holds for all $t \in \mathbb{T}^{\kappa}$.

We then introduce the Cauchy integral or the definite integral by

$$\int_r^s f(\tau)\Delta\tau = F(s) - F(r) \quad \text{for all } r, s \in \mathbb{T}.$$

Definition 5. If $a \in \mathbb{T}$, $\sup(\mathbb{T}) = +\infty$ and f is rd-continuous on $\mathbb{T}_a^+$, then we define the improper integral by

$$\int_a^{+\infty} f(t)\Delta t = \lim_{b \to +\infty} \int_a^b f(t)\Delta t$$

provided this limit exists, and we say that the improper integral converges in this case. Otherwise, we say that the improper integral diverges.

For more details about the delta integral, one can refer to [3, Section 1.4]. We note that the delta integral is studied in terms of Riemann and Lebesgue type integral, as shown in [2, Chapter 5].

Definition 6.
- A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous (denote $f \in \mathcal{C}_{rd} := \mathcal{C}_{rd}(\mathbb{T}, \mathbb{R})$), if
  - ⋆ f is continuous at every right-dense point on $\mathbb{T}$,
  - ⋆ $\lim_{s \to t^-} f(s)$ exists and it is finite at every left-dense point $t \in \mathbb{T}$.
- A function matrix $A : \mathbb{T} \to \mathbb{R}^{n*n}$ is rd-continuous on $\mathbb{T}$, if each entry of A is rd-continuous on $\mathbb{T}$.
- A function $f : \mathbb{T} \times \mathbb{R}^n \to \mathbb{R}^n$ is said to be rd-continuous, if h defined by $h(t) = f(t, x(t))$ is rd-continuous for any continuous function $x : \mathbb{T} \to \mathbb{R}^n$.
- An rd-continuous function $f : \mathbb{T} \to \mathbb{R}$ is said to be regressive (i.e. $f \in \mathcal{R}(\mathbb{T}, \mathbb{R})$) if $1 + \mu(t)f(t) \neq 0$ for all $t \in \mathbb{T}^{\kappa}$. In the following the set $\mathcal{R}(\mathbb{T}, \mathbb{R})$ is denoted $\mathcal{R}$.