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Static feedback control of switched asynchronous sequential machines☆

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ABSTRACT

This paper presents model matching of switched asynchronous sequential machines (ASMs) using static feedback control. Comprised of a number of submachines, the switched ASM changes its present mode according to the switching signal generated by the controller. Not only the state transitions in each submachine but also the switching operations over different submachines are carried out in an asynchronous mechanism. The control goal is to make the stable-state behavior of the closed-loop system exhibit the same input/state characteristics as a given reference model. In particular, we focus on designing a static controller, which consists of only logic gates with no memory elements, in order to minimize the controller size. The study of a synthetic example is provided to illustrate the existence condition for a feedback controller and its construction algorithm.

1. Introduction

Asynchronous sequential machines (ASMs) are referred to as finite-state machines in which the state transition is not governed by a synchronizing clock. Not only do ASMs serve as important building blocks of many engineering systems [1], they have been also used in the modeling of signaling chains in molecular biology [2], mainly due to their clock-less operation formalism. Although their significance does not falter in the field of systems and automation, relatively few studies have been devoted to the control theoretic approach to ASMs. Recently, Yang and Hammer [3,4] propose *corrective control*, a novel automatic control theory for ASMs. Using the feedback control mechanism and asynchrony in ASMs, corrective control compensates the stable-state behavior of existent ASMs without modifying inner logic of the machines. Some interesting results have been achieved in fault-tolerant control of ASMs in both theoretical subjects [3,4] and real-world applications to space-borne digital systems [5,6].

In this paper, we address the switched ASMs and their control problem using the feedback control mechanism. A switched system consists of a collection of subsystems described by linear/nonlinear dynamics along with a rule that elucidates the switching between subsystems. As a wide range of engineering systems in practice can be depicted as switched systems, the study of switched systems – modeling, stability analysis, controllability, etc.– has been attracting much attention in the control academia [7]. We note that compared with rich literature on time-driven switched systems [8], the study of event-driven switched systems is very rare. Among few approaches reported so far are switched Boolean control networks (BCNs), a specific type of switched event-driven systems that are used to describe the model of gene regulatory networks in cells [9,10].

Nonetheless, switched ASMs proposed in this paper represent various dynamical systems. For instance, they can describe asynchronous digital systems with multiple modes that can be switched by some control signals, e.g., bidirectional asynchronous counters [11]. Further, as stated before, biological networks with purely discrete dynamics can be described by switched ASMs if the network dynamics alternate between different switching modes [12]. Note that in all these cases, the present mode, or submachine as is termed in this study, succeeds to the state at which the previous submachine stayed upon being active by the switching signal.

In the switched ASM, each submachine is a single ASM and a feedback controller placed in front of the switched ASM provides the switching signal in response to the external input and the state feedback of the active submachine. The control goal is *model matching*, namely to make the closed-loop system show the same input/state behavior as a reference model. In comparison with switched BCNs, switched ASMs have the unique feature that (i) every state of a submachine can be either stable or transient depending on the present input; and (ii) the asynchronous mechanism is still dominant in the course of switching operations. More specifically, the state of switched BCNs evolves only once at

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each switching operation. On the other hand, switched ASMs may undergo multiple state transitions by the switching signal as no global synchronizing clock governs the system dynamics. As will be discussed later, these characteristics impose a constraint on the generation of the switching signal.

The feedback controller developed in our study also differs from the corrective controller for a single ASM [3–6] in the following aspects. First, while the corrective controller has the form of a dynamic ASM, the feedback controller in our study will be designed as *static*, consisting only of logical gates (or modules) and requiring no memory elements, in other words, it is simplest in design perspective. Next, whereas the corrective controller is able to change the input character to the ASM by its control law, the feedback controller for the switched ASM solely produces the switching signal and does not change the external input. Hence the control objective must be accomplished only by setting an appropriate switching law, as the external input is delivered to the switched ASM without modification.

The remainder of the paper is organized as follows. Section 2 introduces a model of switched ASMs and the problem statement. In Section 3, we address the reachability of the switched ASM in a matrix representation, and present the existence condition for a static feedback controller that solves a given model matching problem, as well as a detailed design procedure for the controller. In Section 4, the effectiveness of the result is demonstrated through an illustrative example. Finally, some concluding remarks are given in Section 5.

In the companion paper [13], we explore the issue of synthesizing *dynamic* feedback controllers for switched asynchronous machines. In [13], the control objective is also model matching, but more resource is needed in designing the controller since it has states.

2. Preliminaries

2.1. Switched ASMs

We describe a switched ASM Σ with m submachines as

$$\Sigma = \{\Sigma_i | i \in M\}, \quad \Sigma_i = (A, X, x_0, f_i) \tag{1}$$

where $M = \{1, \ldots, m\}$, Σ_i is an input/state ASM that represents the ith submachine, A and X are the input and state set, respectively, x_0 is the initial state, and $f_i : X \times A \to X$ is the state transition function of Σ_i. As all the submachines are building blocks of Σ, they have the same input and state set and the initial state, whereas their transition functions f_i's are different with each other.

The behavior of Σ_i complies with the characteristics of a single ASM. A valid state and input pair $(x, \upsilon') \in X \times A$ is a *stable combination* if $f_i(x, \upsilon') = x$. Σ_i lingers at x indefinitely as long as the input υ' remains fixed. If the input changes to another value υ such that $f_i(x, \upsilon) \neq x$, (x, υ) is a *transient combination* of Σ_i. (x, υ) gives rise to a chain of transitions $x_1 = f_i(x, \upsilon)$, $x_2 = f_i(x_1, \upsilon), \ldots$, in which the input υ remains the same. Assuming no infinite cycles, Σ_i reaches the *next stable state* $x' = f_i(x', \upsilon)$ and stays there waiting for further change of the input. Due to the absence of a synchronizing clock, the transient transitions in an ASM are so instantaneous that the transient states $x_1, x_2, \ldots$ are unnoticeable to outer users. Hence, it is proper to express state transitions solely in terms of stable states. To this end, we define the *stable recursion function* $s_i : X \times A \to X$: for every valid pair $(x, \upsilon) \in X \times A$, $s_i(x, \upsilon) := x'$, where x' is the next stable state of (x, υ). A chain of transient transitions from a stable state to its next stable state, as represented by s_i, is termed a *stable transition*. s_i can be extended to input strings by defining $s_i(x, \upsilon_1\upsilon_2 \cdots \upsilon_k) := s_i(s_i(x, \upsilon_1), \upsilon_2 \cdots \upsilon_k)$ for an input string $\upsilon_1\upsilon_2 \cdots \upsilon_k \in A^+$. For later usage, let $U_i(x) \subset A$ and $T_i(x) \subset A$ be the set of inputs that make a stable and transient combination with the state x in Σ_i, respectively.

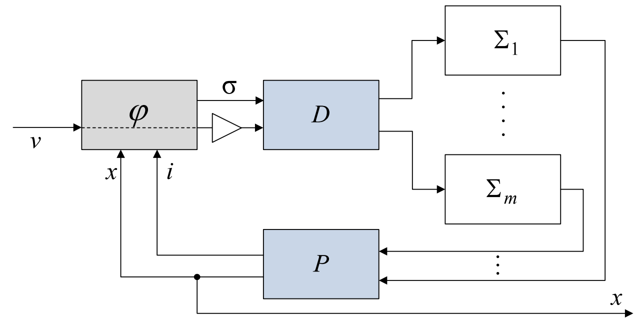

Fig. 1. Static feedback control system for the switched ASM.

2.2. Closed-loop system

Fig. 1 is the architecture of the proposed static feedback control system for the switched ASM Σ. φ is the static feedback controller and D and P symbolize the *demultiplexer* and *multiplexer*, respectively. $\upsilon \in A$ is the external input and $\sigma \in M$ is the switching signal provided by φ. D adjusts the input channel depending on σ, that is, it delivers υ to submachine Σ_σ. Hence the change of σ characterizes the switching operation. P collects the state feedback of all submachines among which $x \in X$, the value of the active submachine, is provided to φ along with the index of the active submachine $i \in M$.

Since φ is the static feedback controller, it is represented by a function $\varphi : M \times X \times A \to M$ that generates the switching signal σ from the external input υ and the feedback (i, x) according to the relation

$$\varphi(i, x, \upsilon) = \sigma. \tag{2}$$

We denote by Σ_φ the closed-loop system composed of φ and Σ. As marked in Fig. 1, υ is the read-only variable of φ. It will be sent to D via φ without any modification.

Owing to the lack of a synchronizing clock, one must design an ASM in such a way that no two variables change simultaneously so as to avoid unpredictable outcome. This policy, termed the principle of the fundamental mode operation [14], has to be also preserved by Σ_φ. To this end, we lay down the following operation rules for φ and Σ, part of which are derived from the prior works [3].

Rule 1. Fundamental mode operations for switched ASMs:

1. σ and υ must change their values only when Σ stays at a stable state, and only one at a time. On the contrary, if σ or υ changes while Σ undergoes transient transitions, the next stable state would be unspecified due to asynchrony and the rapid speed of transitions.
2. To prevent a simultaneous change of σ and υ, φ is designed so that the generation of σ precedes the transmission of υ in φ. For instance, assume that Σ has stayed at a stable combination (x, υ) of submachine Σ_i, when the external input changes to υ'. φ then provides D with the new switching signal $j = \varphi(i, x, \upsilon')$ while υ' is not delivered to D yet. If $j \neq i$, Σ undergoes the switching from Σ_i to Σ_j. Note that the input to Σ remains υ throughout the switching operation. Not until the completion of the switching operation does the input change to υ'.

Rule 1.2 can be implemented by inserting a buffer in the input channel as shown in Fig. 1. Since φ is designed using only combinational logic gates, we can ensure that the switching signal will be transmitted to Σ ahead of the changed external input.