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A receding horizon stabilization approach to constrained nonholonomic systems in power form

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ABSTRACT

The control of nonholonomic systems with practical requirements is still a challenging problem but finds many industrial applications. This paper studies the receding horizon control (RHC)-based stabilization problem of a class of constrained nonholonomic systems in power form. A non-quadratic cost function is constructed by using the homogeneous norm of the nonholonomic system in power form. With this novel cost function, two kinds of RHC algorithms are designed, of which one ensures the convergence of the closed-loop system states, and the other ensures the ρ-exponential stability. The feasibility of the designed algorithms and the closed-loop convergence are analyzed and ensured theoretically under mild conditions. The comparison and application results are provided, showing that (1) the proposed RHC algorithms are effective and the theoretical results are valid, and (2) the proposed algorithms can stabilize the nonholonomic systems with a much faster convergence rate than the conventional time-varying stabilizable controllers.

1. Introduction

Due to the fact that most of the mechanical systems (such as automobile, robots, unmanned aerial vehicles (UAVs) and under-actuated autonomous underwater vehicles (AUVs)) involve nonholonomic constraints, the control of nonholonomic systems is of particular industrial interest, and has been an active topic in last three decades [1–4]. The challenge of controlling nonholonomic systems is caused by the nonholonomic constraints, resulting in that the linearized system at the origin losses controllability and there does not exist a locally static continuous state feedback controller to stabilize such nonlinear systems; see the Brockett's theorem in [5]. Many results have been proposed to solve the stabilization and tracking problems of nonholonomic systems, including the discontinuous feedback techniques [6–8], the time-varying feedback controllers [9–11] and recently, the switched and hybrid control strategies [1,3].

In practice, the control inputs of a mechanical system are likely to be constrained due to actuator saturations, and the control performance is expected to be optimized. Several results have been reported for the stabilization problem of nonholonomic systems addressing these practical issues. For example, in [12,13], the stabilization of nonholonomic systems in chained form are addressed by considering practical control input bounds. To obtain optimal control performance, Qu et al. investigate the stabilization problem of nonholonomic systems in chained form based on LQR and a near-optimal control performance is achieved in [14]. However, none of the aforementioned results consider the control input constraints and optimal performance simultaneously.

It is well-known that the receding horizon control (RHC), also known as model predictive control enjoys wide popularity due to its capability of handling system constraints, achieving (sub)-optimal control performance, and dealing with nonlinear dynamics; see [15–20] for example. In addition, the RHC provides piece-wise control law which may offer a feasible solution to the stabilization problem of nonholonomic systems. Motivated by these facts, in this study, we investigate the RHC-based strategy for the stabilization problem of a class of nonholonomic systems in power form with control input constraints. Note that most of the nonholonomic systems can be transformed into power forms through appropriate coordinate transformation [21], and that the nonholonomic systems in power forms are equivalent to the chained forms [22].

It is worth noting that the design and analysis of RHC strategy for nonholonomic systems are non-trivial. Due to the fact that there does not exist a stabilizable state feedback controller for the linearized system of the nonholonomic systems at the origin, the assumption that there exists a locally linear state feedback law in the conventional RHC [16,17,23,24] does not hold, and thus these RHC strategies cannot be applied. In [25], a general frame for RHC design is proposed for nonlinear systems, however, it

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is generally very hard and unclear how to construct a terminal region with a stabilizable controller as in Condition S5); see also the last example given in [25]. In [26], the RHC strategy is developed for the stabilization problem of a wheeled vehicle by using quadratic cost functions, but the feasibility is not analyzed and theoretically guaranteed. Except these results, most of the RHC strategies for nonholonomic vehicles (e.g., [27–30]) are application results without rigorous analysis and performance guarantee. As a result, it is very necessary to develop a general RHC framework for constrained nonholonomic systems with theoretically guaranteed performance.

In this paper, we propose a general approach to designing the RHC algorithms for a class of constrained nonholonomic systems in power form, and provide theoretical analysis for the parameter design with guaranteed feasibility and closed-loop convergence. The RHC algorithm design makes use of the homogeneity of the nonholonomic systems and the time-varying controller in [31]. The main contributions of this paper are three-fold:

- A general RHC-based approach to the stabilization of constrained nonholonomic systems in power forms is proposed. A novel cost function is first constructed, and this cost function is not in conventional quadratic form, but a function of the homogeneous norm of the nonholonomic systems with fixed structure. Built on the cost function, two novel RHC algorithms, with one ensuring state convergence and the other ensuring ρ-exponential stability are designed.
- The theoretical results on guaranteeing control performance under the proposed RHC approach are provided. It is shown that under the way of designing the terminal set, the designed RHC algorithms are iteratively feasible. In addition, we show that, under the same conditions, the closed-loop systems are guaranteed to be convergent and ρ-exponentially stabilized, respectively, by using the two RHC algorithms. These theoretical results provide a general approach to parameter design, leading to feasible and stable RHC algorithms in practical implementation.
- The comparison study and application of the designed RHC algorithm are provided. Comparison results show that the designed RHC algorithms ensure constraint fulfillment, and stabilize the closed-loop system with a much faster convergence rate than the conventional time-varying exponential stabilization strategy in [31]. The application procedure of the proposed RHC for stabilization of a constrained nonholonomic car is presented, including a coordinate transformation and application of the proposed RHC. The application example verifies the potential applicability of the proposed approach.

The remainder of this paper is structured as follows. The problem formulation and preliminaries on homogeneous systems are presented in Section 2. The RHC algorithm ensuring state convergence is designed in Section 3, and the feasibility and closed-loop convergence analysis is presented Section 4. The results on the RHC algorithm with ρ-exponential stability are provided in Section 5. In Section 6, a simulation and comparison study is conducted, and in Section 7, the application result to a nonholonomic car is presented. Finally, the conclusion is summarized in Section 8.

The notations used in this paper are as follows. The superscript "T" denotes the matrix transposition. We use the symbol $\mathbb{R}$ to denote the real number, and $\mathbb{R}^n$ to denote an n-dimensional real space. Given a column vector $x \in \mathbb{R}^n$, denote its Euclidean norm as $|x|$. Given two sets $\mathcal{U} \in \mathbb{R}^n$ and $\mathcal{W} \in \mathbb{R}^n$ with $\mathcal{U} \subseteq \mathcal{W}$, by $\mathcal{W} \setminus \mathcal{U}$, we mean that the set $\{x | x \in \mathcal{W}, x \notin \mathcal{U}\}$. The symbol $\mathrm{col}\{x_1, x_2, \ldots, x_n\}$ denotes the column operation as $[x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \ldots, x_n^{\mathrm{T}}]^{\mathrm{T}}$ for column vectors $x_1, x_2, \ldots, x_n$.

2. Problem formulation and preliminaries

2.1. Problem formulation

Consider the following nonholonomic systems described in power form:

$$\dot{x}_1(t) = u_1$$
$$\dot{x}_2(t) = u_2$$
$$\dot{x}_i(t) = \frac{x_2^{i-2}}{(i-2)!} u_2, \quad i = 3, \ldots, n, \tag{1}$$

where x_i are the system states, and u_j, $j = 1, 2$ are the control inputs. The control inputs are required to satisfy the constraints as

$$u_j \in \mathcal{U}_j, \quad j = 1, 2, \tag{2}$$

where $\mathcal{U}_j \subseteq \mathbb{R}$ and contain zero as their interior point. The system in (1) can also be written as

$$\dot{x} = f(x, u),$$

where $x = \mathrm{col}(x_1, \ldots, x_n)$, and $u = \mathrm{col}(u_1, u_2)$. The objective of this study is to design a RHC algorithm such that the closed-loop system in (1) is stable with respect to the equilibrium point and the constraints in (2) fulfill.

Note that even without the control input constraints, there does not exist a state feedback control law locally stabilizing the system in (1), due to the fact that the linearized system at zero is not controllable. This is a direct result caused by nonholonomic property of the system in (1); see [5] for detailed reason. As a result, the assumption on the existence of a linear state feedback controller in [16,17,23,24,32] is not satisfied, and these popular RHC algorithms are not applicable for the system in (1).

In this study, we will make use of the homogeneity of the system in (1) to design the RHC strategies.

2.2. Preliminaries

In order to make use of the homogeneity of the system in (1), a few definitions on dilation, homogeneous function, norm and vector field are recalled and presented [31,33].

Definition 1 (*Dilation and Homogeneous Norm*)**.** For $x \in \mathbb{R}^n$, a dilation operation on $\mathbb{R}^n$ is defined by the following map $\delta_\lambda^r : \mathbb{R}^n \mapsto \mathbb{R}^n$:

$$\delta_\lambda^r = (\lambda^{r_1} x_1, \lambda^{r_2} x_2, \ldots, \lambda^{r_n} x_n), \lambda > 0,$$

where $r = (r_1, \ldots, r_n)$ with $r_i \geqslant 1$. A homogeneous norm associated with the dilation δ_λ^r is defined as

$$\rho_p(x) = \left(\sum_{i=1}^{n} |x_i|^{p/r_i} \right)^{1/p}, p > 0.$$

Definition 2 (*Homogeneous Function and Vector Field*)**.** A continuous function $f : \mathbb{R}^n \times \mathbb{R} \mapsto \mathbb{R}$ is said to be homogeneous of degree $l \geqslant 0$ with respect to the dilation δ_λ^r, if

$$f(\delta_\lambda^r x, t) = \lambda^l f(x, t), \forall \lambda > 0.$$

In addition, a continuous vector field $g : \mathbb{R}^n \times \mathbb{R} \mapsto \mathbb{R}^n \times \mathbb{R}$ is said to be homogeneous of degree $l \geqslant 0$ with respect to the dilation δ_λ^r, if

$$g_i(\delta_\lambda^r x, t) = \lambda^{l+r_i} g_i(x, t), \forall i = 1, \ldots, n.$$