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A travelling wave approach to a multi-agent system with a path-graph topology[☆]

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ABSTRACT

The paper presents a novel approach for the analysis and control of a multi-agent system with non-identical agents and a path-graph topology. With the help of irrational wave transfer functions, the approach describes the interaction among the agents from the 'local' perspective and identifies travelling waves in the system. It is shown that different dynamics of the agents create a virtual boundary that causes a partial reflection of the travelling waves. Undesired effects due to the reflection of the waves, such as amplification/attenuation, long transients or string instability, can be compensated by the feedback controllers introduced in this paper. We show that the controllers achieve asymptotic and even string stability of the system.

1. Introduction

A path graph is one of the simplest and most studied interaction topologies of a multi-agent system. This topology is used in many applications, such as vehicle platoons [1,2], discretized flexible structures [3,4], or spatially-discretized models of long electrical transmission lines [5]. Formally, a path graph is a graph with N vertices, ordered as $v_1, v_2, \dots, v_N$, with edges between vertices $\{v_i, v_{i+1}\}$, $i = 1, \dots, N-1$. Equivalently, in a path graph topology, each agent, except for the first and last one, interacts with its two neighbours (see Fig. 1).

There are many tools for describing multi-agent systems. They range from state-space techniques [6], polynomial approaches [7] to statistical-physics-based description [8]. In path graphs, scaling of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms was calculated in [1] and [9]. Boundedness of the norm of interest for any N can be captured by a term "string stability". Roughly speaking, in a string-stable system a disturbance is not amplified as it propagates among the agents (see [10] for various definitions). The approaches mentioned so far are very useful in analysing aggregate properties of the multi-agent system, such as its stability or system norms. On the other hand, it is more difficult to infer from them what happens in the middle of the system or near the boundaries.

Following the ideas of other researchers (e.g., [11,12]), we will describe the system using a wave perspective. Indeed, the propagation of the change in the multi-agent system can be described with the help of travelling waves. We will illustrate it on an example of a system with identical agents and a path-graph topology. If the first agent changes its output, then all following agents sequentially respond to this change. If we study their response from the local point of view [13–15] we can notice that the change is propagated as a wave. The wave departs from the first agent and travels along the system to the last agent, where it reflects and travels back. When it reaches the first agent, it reflects again. A similar phenomenon is apparent when the agents are non-identical—the travelling wave is partially reflected on non-identical agents [16]. We can imagine this behaviour as the reflection of the wave if it encounters a boundary between two media of different properties.

The tool for analysis in this paper will be so called *wave transfer function* (WTF). The transfer-function approach to waves has recently been revisited in a series of papers for lumped models [13,17] and for continuous flexible structures [15]. The travelling wave approach has also been applied to vibration control [18] and it seems to be related to the impedance matching in the power networks [19]. The wave-based description leads to irrational transfer functions, analysis of which differs in several aspects from their rational counterparts [20].

This paper continues in the research started in [14], where waves in a platoon of identical vehicles were considered. A natural extension of this model is to consider a chain of non-identical (heterogeneous) agents. The first step in the treatment of non-identical agents using travelling waves is given in [21] for a mass–spring model. We generalize it by considering arbitrary dynamics of the

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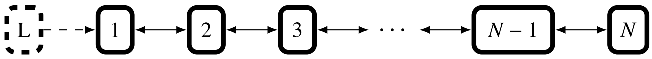

Fig. 1. Structure of the path graph. The arrows show interaction between agents. The agent L is a virtual leader of the multi-agent system, which commands only the first agent.

agents and their controllers. The preliminary results are presented in [16], where we introduced the *soft boundary* in a chain of vehicles. Here, we introduce the second fundamental type of boundary, the *hard boundary*. Although the boundaries are virtual in nature, they principally affect the overall system behaviour. We present some fundamental properties of the boundaries and design wave-absorbing controllers for both types of boundaries.

The main contributions of the paper are: (i) mathematical description of the travelling waves in a multi-agent system with non-identical agents given by Theorems 1 and 2, (ii) a design of a controller that prevents a reflection of the travelling wave (Theorems 3 and 4) and (iii) proof of stability and string stability when these controllers are used (Theorem 5). For better understanding and easy simulations, we provide a set of functions in MATLAB, see *WaveBox* [22].

2. System model

In the whole paper we work only with LTI systems in the Laplace domain, all transfer functions are SISO and signals are assumed to be scalars. The argument (s) denotes the Laplace variable and can be omitted when no ambiguity seems possible.

We consider a multi-agent system of N non-identical agents with a path-graph interaction topology. Each agent interacts with its nearest neighbours using output feedback. The LTI dynamics of the agent i consists of two parts: the model of the plant $P_i(s)$ and the models of the controllers. The controller $C_i^{\mathrm{L}}(s)$ processes the output error $X_{i-1}(s) - X_i(s)$ to agent's predecessor $i-1$ and the controller $C_i^{\mathrm{R}}(s)$ processes the output error $X_{i+1}(s) - X_i(s)$ to agent's follower $i+1$. Based on the order of agents in Fig. 1, we denote the interaction of the agent i with agent the $i-1$ by superscript 'L' (*left* of agent i) and the interaction with agent $i+1$ by superscript 'R' (*right* of agent i). The input $U_i(s)$ of the plant is generated by the controllers as

$$\begin{aligned} U_i(s) = {} & C_i^{\mathrm{L}}(s)\left(X_{i-1}(s) - X_i(s) + W_i^{\mathrm{L}}(s)\right) \\ & + C_i^{\mathrm{R}}(s)\left(X_{i+1}(s) - X_i(s) + W_i^{\mathrm{R}}(s)\right), \end{aligned} \quad (1)$$

where $W_i^{\mathrm{L}}(s)$ and $W_i^{\mathrm{R}}(s)$ are external inputs to the agent, which will be defined later on. The output of the agent is then given by $X_i(s) = P_i(s)U_i(s)$. By defining the *left* open-loop transfer function (OLTF) $M_i^{\mathrm{L}}(s) = P_i(s)C_i^{\mathrm{L}}(s)$ and *right* OLTF $M_i^{\mathrm{R}}(s) = P_i(s)C_i^{\mathrm{R}}(s)$, we obtain the overall model of the agent

$$\begin{aligned} X_i(s) = {} & M_i^{\mathrm{L}}(s)\Big(X_{i-1}(s) - X_i(s)\Big) + M_i^{\mathrm{R}}(s)\Big(X_{i+1}(s) - X_i(s)\Big) \\ & + M_i^{\mathrm{L}}(s)W_i^{\mathrm{L}}(s) + M_i^{\mathrm{R}}(s)W_i^{\mathrm{R}}(s). \end{aligned} \quad (2)$$

The structure of the ith agent is depicted in Fig. 2. Usually, there is at least one integrator both in the left and right OLTFs (for instance, from velocity to position), such that the OLTF can be factored as $M(s) = 1/s^{\nu}\overline{M}(s)$, where $\overline{M}(0) < \infty$ and ν is the number of integrators in the corresponding open loop (either M_i^{L} or M_i^{R}).

Often, the system has a reference X_{ref} which it should track. This reference is given by a (virtual) leader, which is usually connected to one of the end nodes. Without loss of generality, we assume that it is connected to the first agent, which is then described as $X_1(s) = M_1^{\mathrm{L}}(s)(X_{\mathrm{ref}}(s) - X_1(s) + W_1^{\mathrm{L}}(s)) + M_1^{\mathrm{R}}(s)(X_2(s) - X_1(s) + W_1^{\mathrm{R}}(s))$. The last agent $i = N$ has only one neighbour, so its model is $X_N(s) = M_N^{\mathrm{L}}(s)\big(X_{N-1}(s) - X_N(s) + W_N^{\mathrm{L}}(s)\big)$.

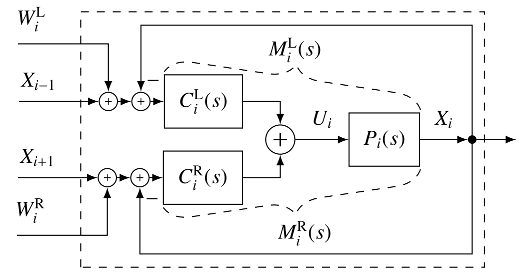

Fig. 2. The model of ith agent.

2.1. Wave transfer function

The key idea of the wave approach is that the output of the ith agent is decomposed into two components, $A_i(s)$ and $B_i(s)$ such that $X_i(s) = A_i(s) + B_i(s)$. The component $A_i(s)$ represents a wave which propagates from left to right, that is, to the agents with higher indices. The component $B_i(s)$ represents the wave propagating from right to left—to the agents with lower indices. The idea is similar to the standard D'Alambert solution of the wave equation in PDE, where also two waves propagating in different directions appear [23].

Now we summarize the results of [14], where we considered identical agents. In this case $M_i^{\mathrm{L}}(s) = M_i^{\mathrm{R}}(s) = M(s)$. The *Wave transfer function* $G(s)$ captures how the wave propagates in the system in one direction, that is $A_{i+1}(s) = G(s)A_i(s)$ and $B_{i-1}(s) = G(s)B_i(s)$. There is a simple way how to derive this transfer function. Consider a path graph with infinite number of agents and with only a wave propagating in the direction of increasing index. Since there is no end in this system, the wave will never reflect back, so $X_i(s) = A_i(s)$. Then WTF is given by $G(s) = X_{i+1}(s)/X_i(s)$ for $N \to \infty$, see [14, Sec. 3.1]. When $W_i^{\mathrm{L}} = W_i^{\mathrm{R}} = 0$, the system with identical agents is described for $i \in [1, N-1]$ as

$$X_i(s) = A_i(s) + B_i(s), \quad (3)$$

$$A_{i+1}(s) = G(s)A_i(s), \quad (4)$$

$$B_i(s) = G(s)B_{i+1}(s), \quad (5)$$

$$G(s) = \frac{1}{2}\alpha(s) - \frac{1}{2}\sqrt{\alpha^2(s) - 4}, \quad (6)$$

where $\alpha(s) = 2 + 1/M(s)$, or, alternatively, $\alpha(s) = G(s) + G^{-1}(s)$. The function $G^{-1}(s) = 1/G(s) = \frac{1}{2}\alpha(s) + \frac{1}{2}\sqrt{\alpha^2(s) - 4}$. We now explain the travelling wave concept. Combining (3)–(5),

$$X_i(s) = G(s)A_{i-1}(s) + G(s)B_{i+1}(s). \quad (7)$$

This means that the wave $A_{i-1}(s)$ coming from the left (from the agent with lower index) is transformed through the transfer function $G(s)$ and summed with the transformed wave $G(s)B_{i+1}(s)$ from the right (from the agent with higher index).

Now suppose that the number of agents is finite. Then, as we discussed in [14], there are two types boundaries in the homogeneous system, located at the end nodes in the path graph. The *forced-end boundary* is caused by the leader's output X_{ref}. If the leader affects the first agent, the boundary is described by

$$A_1(s) = G(s)X_{\mathrm{ref}}(s) - G^2(s)B_1(s). \quad (8)$$

The *free-end boundary* is at the end node which has only one neighbour. If it is located at the Nth agent, it is given by

$$B_N(s) = G(s)A_N(s). \quad (9)$$

The first type of the boundary is analogous to Dirichlet boundary condition ("zero position") and the second to Neumann ("zero derivative with respect to position") [23]. The signal propagation in the system with boundaries is shown in Fig. 3.